Lecture 4

BST: Insertion & Deletion, Intro to Red-Black Trees

Insertion in a BST

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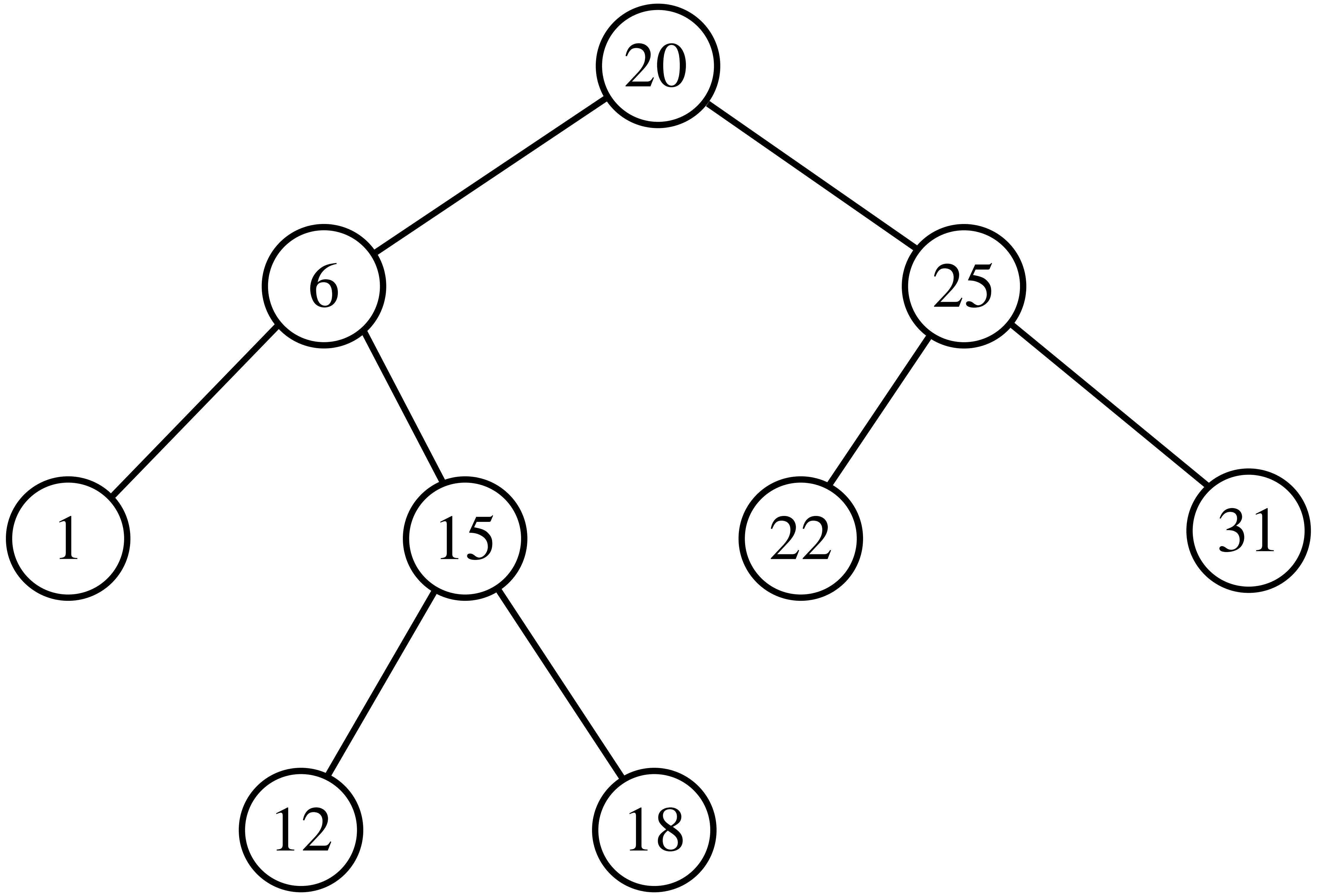
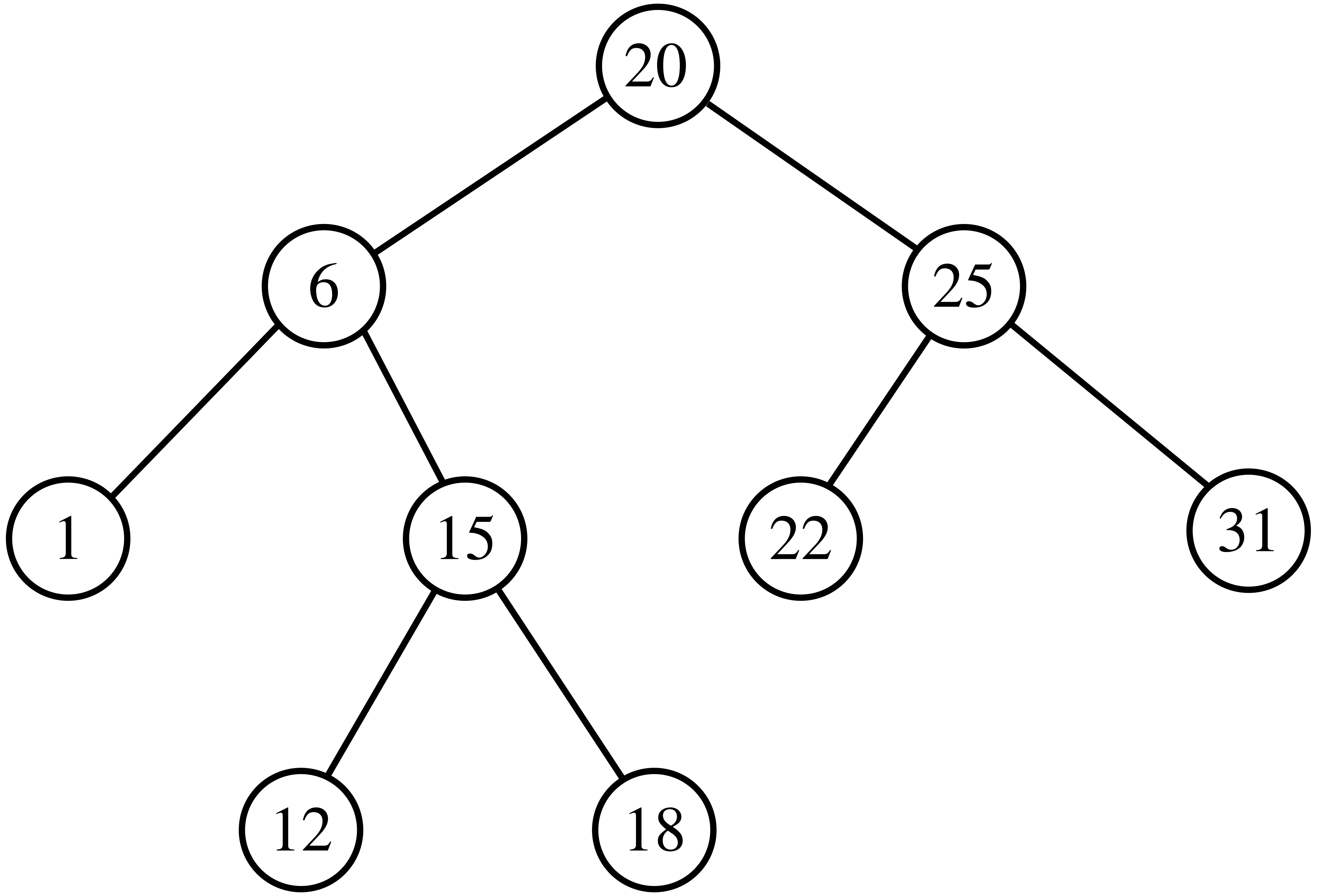


Insertion in a BST

Example:



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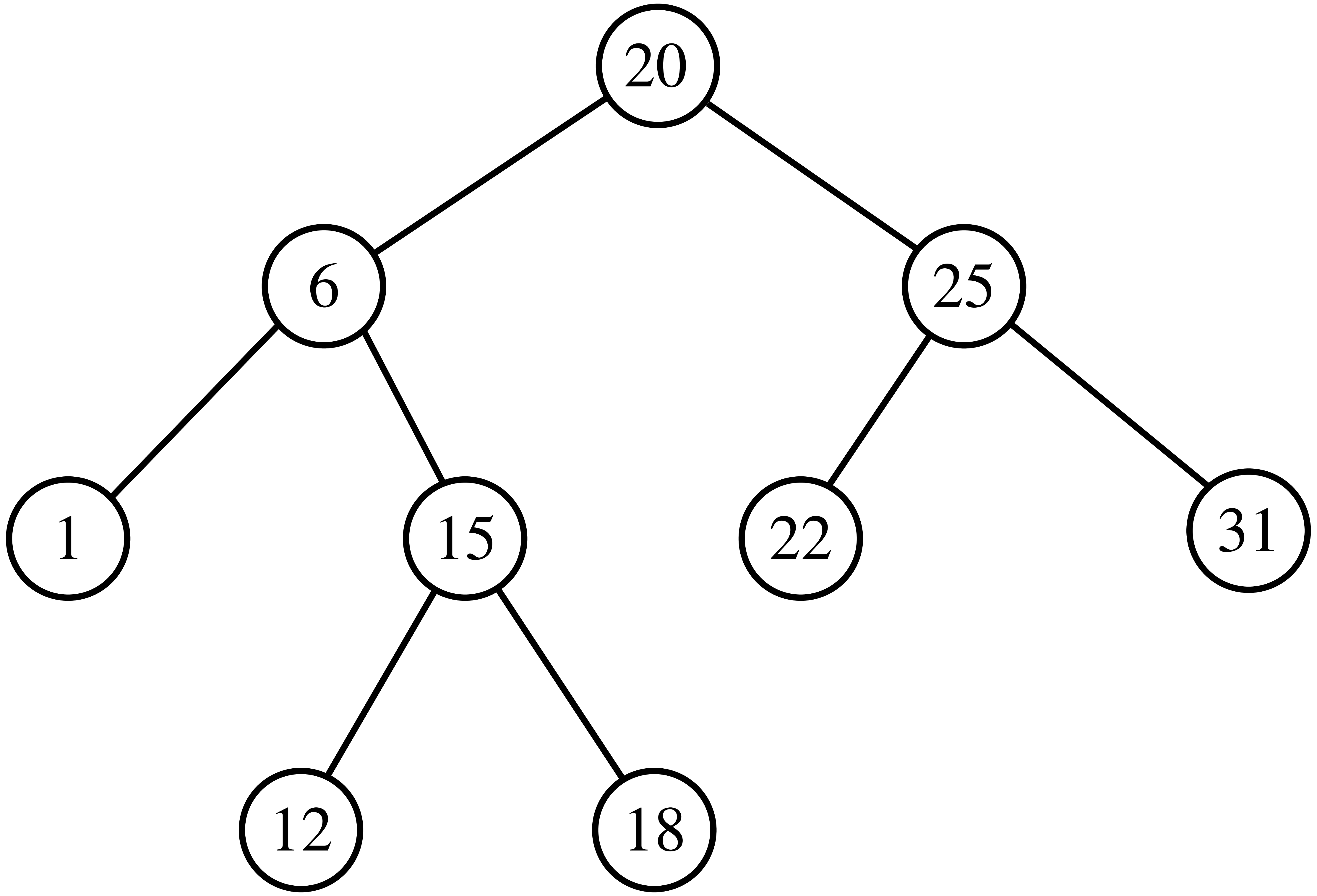
Example: Insert a node with 24 as key in the following BST.



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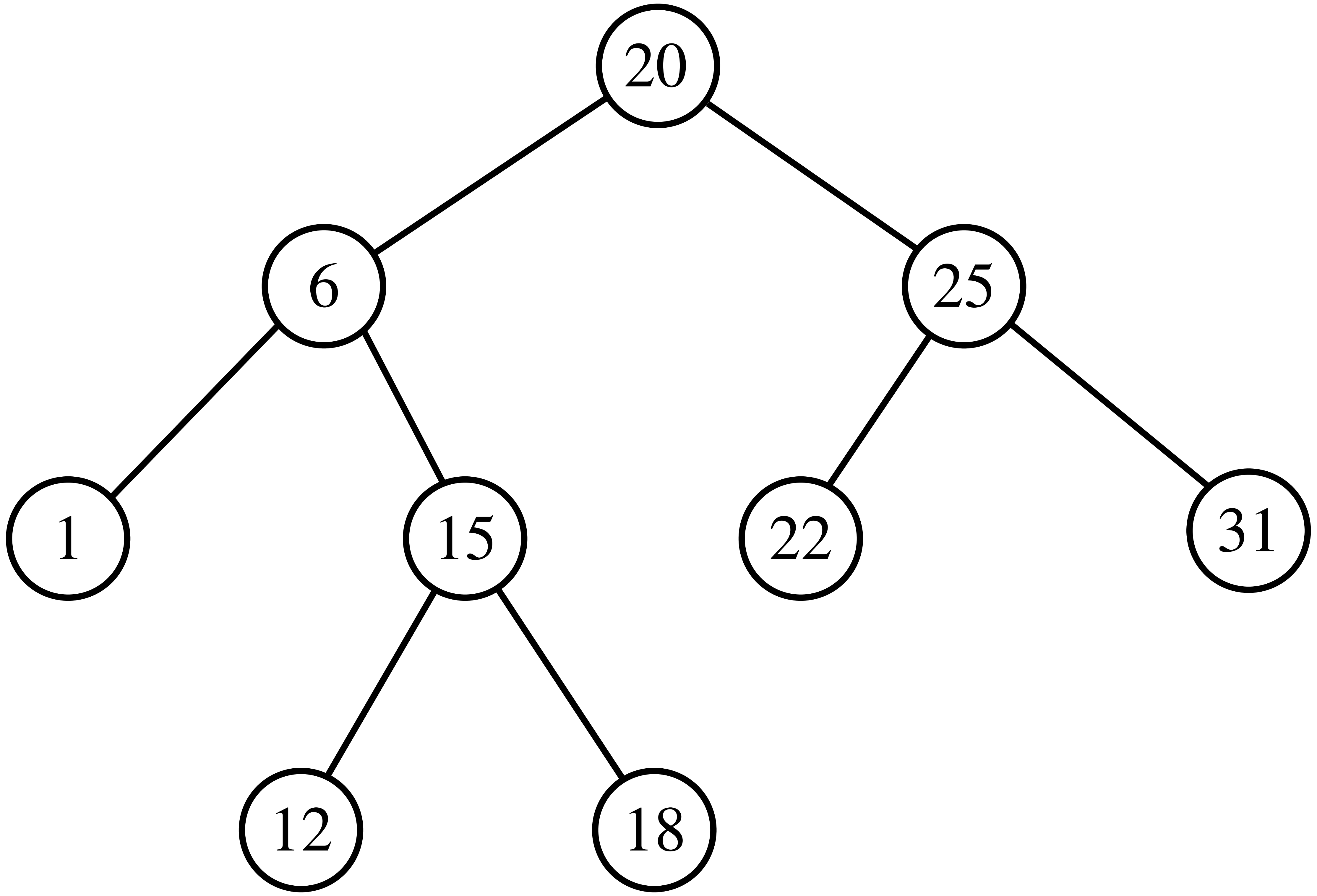

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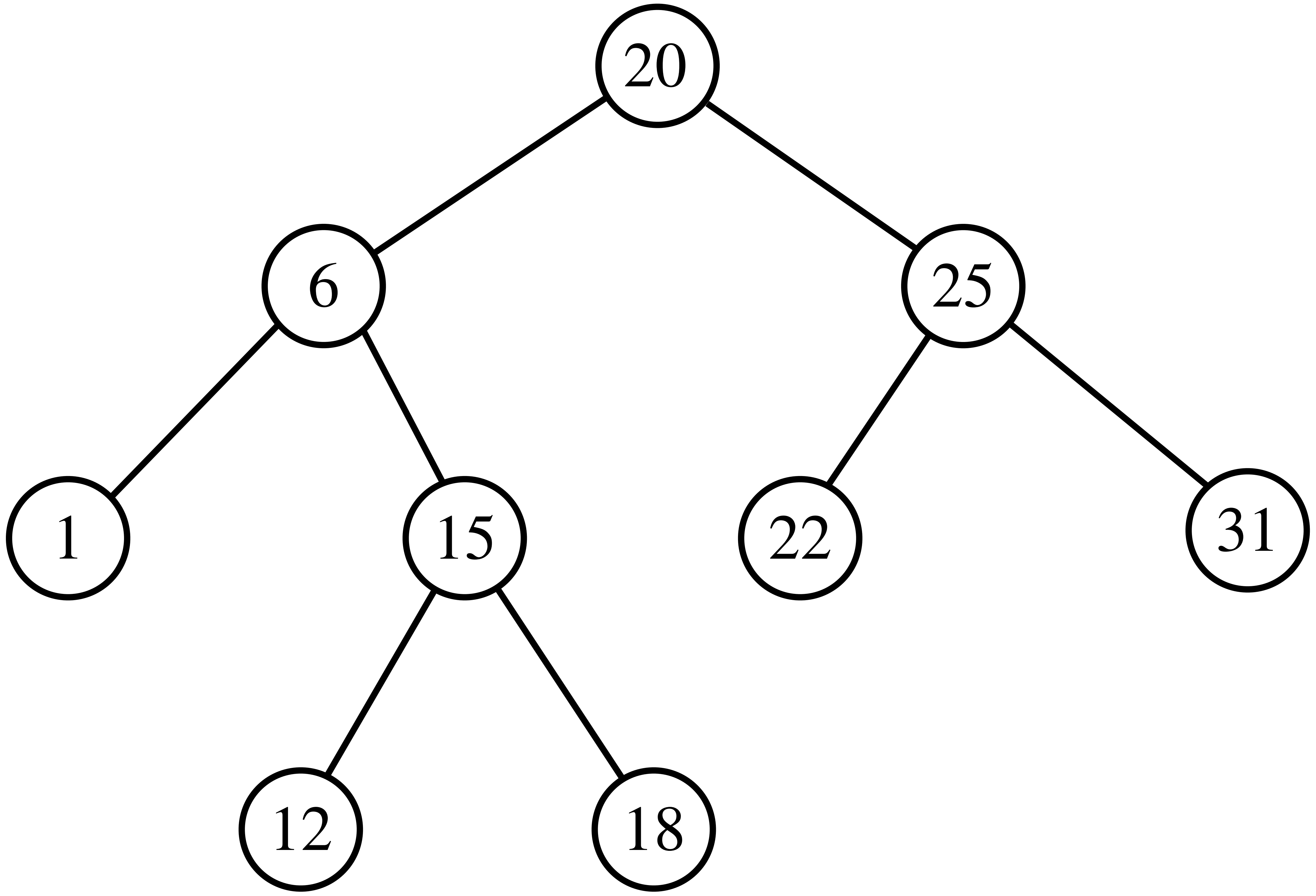
Idea: Find the correct leaf where it can be inserted.



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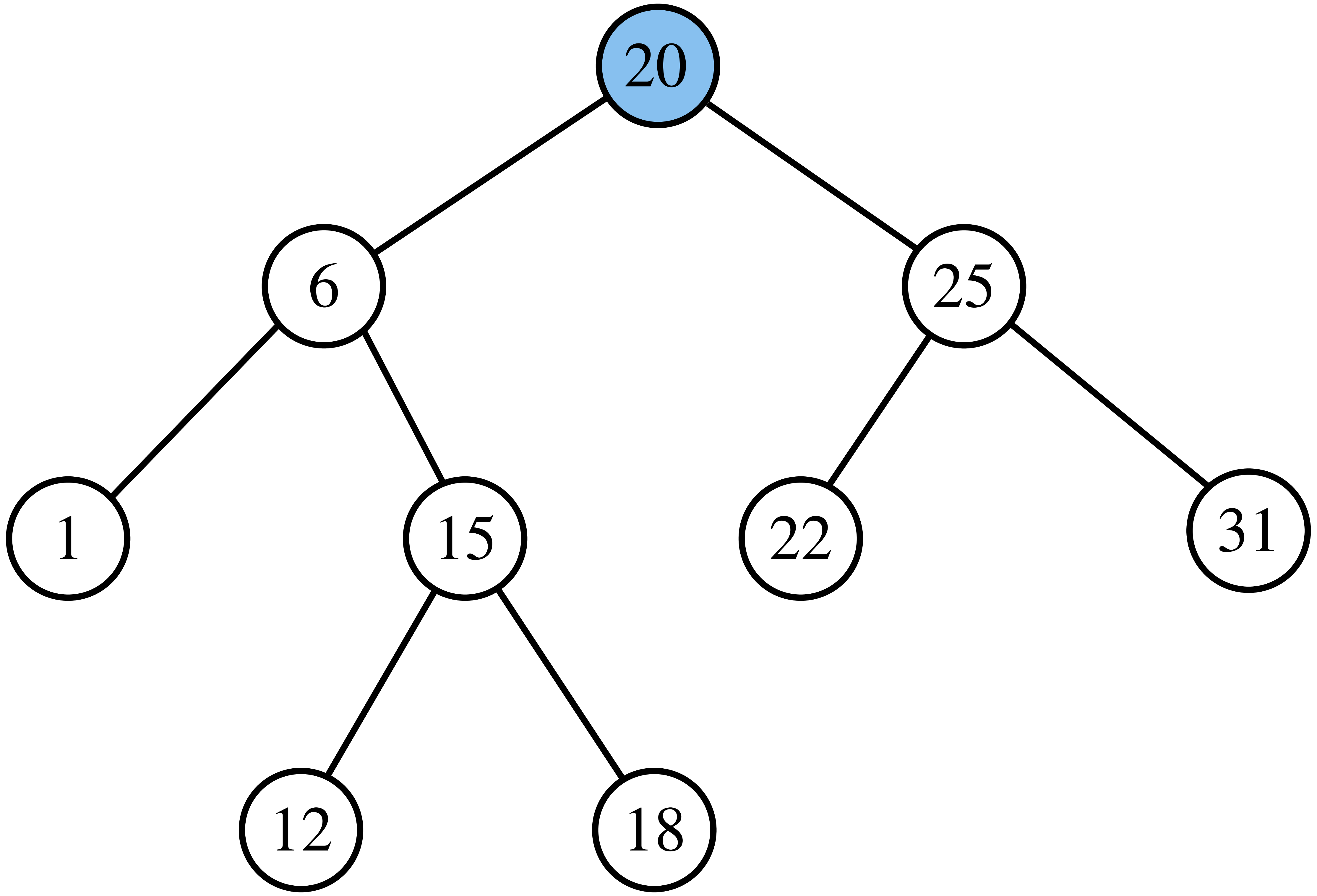

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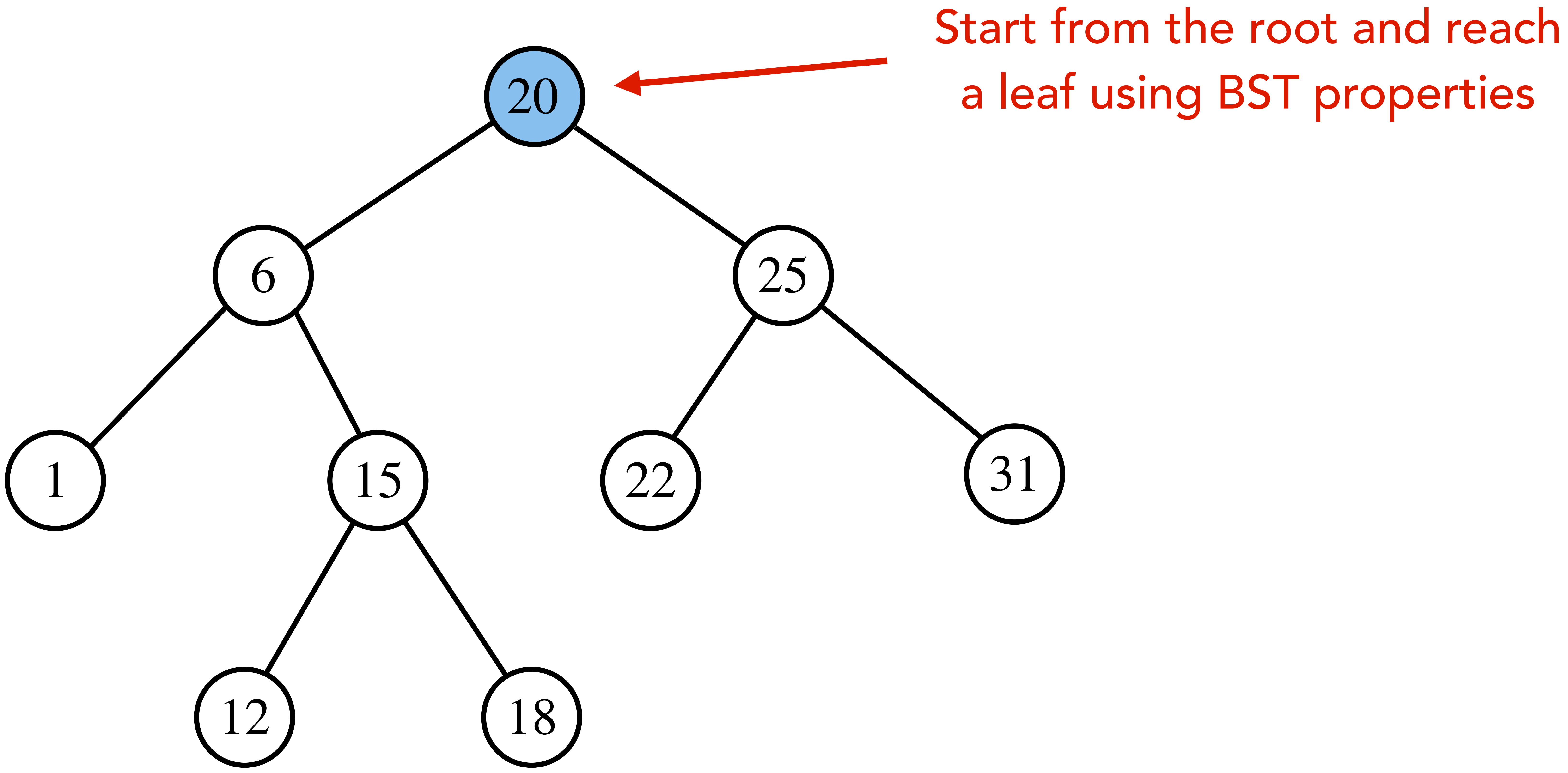

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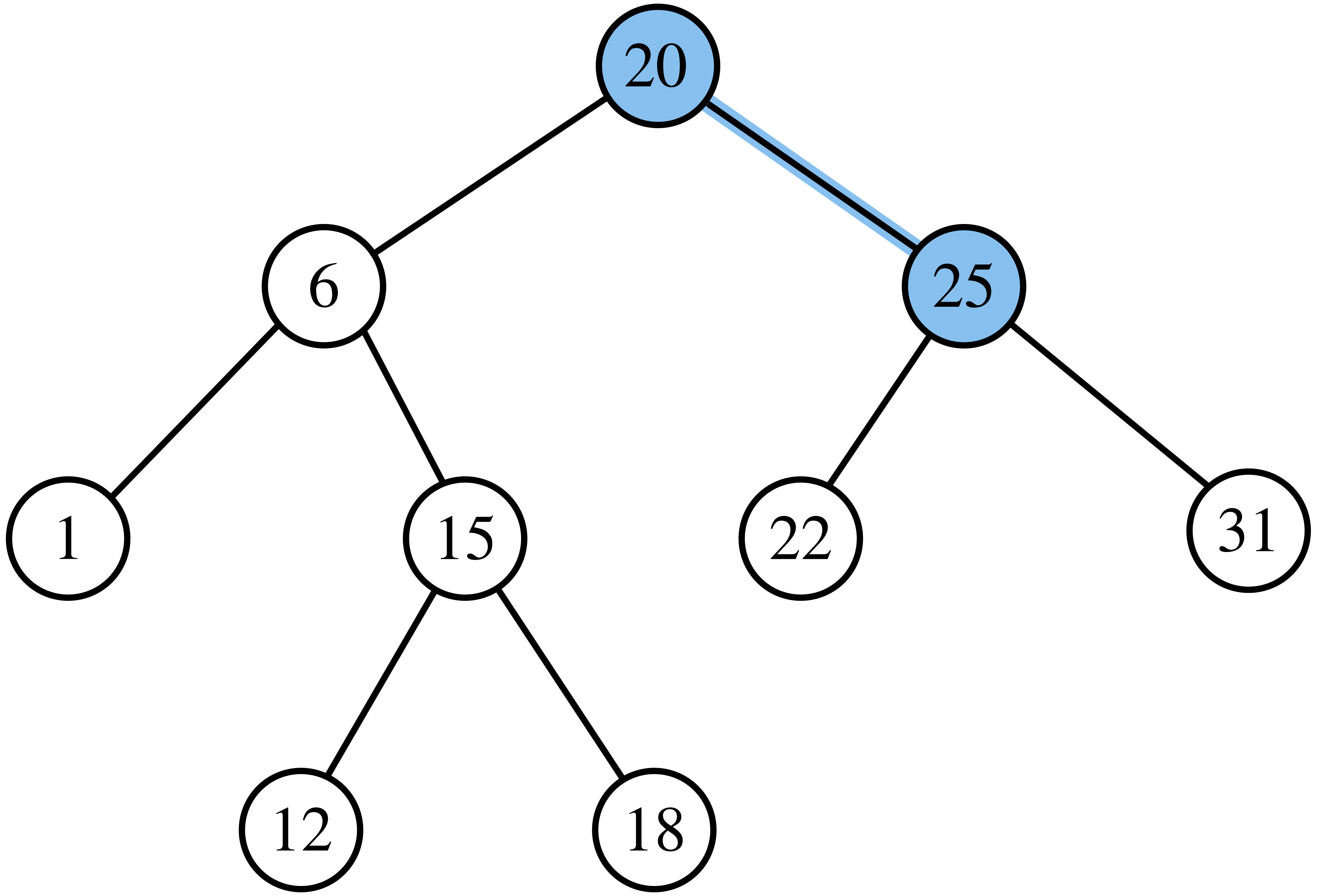

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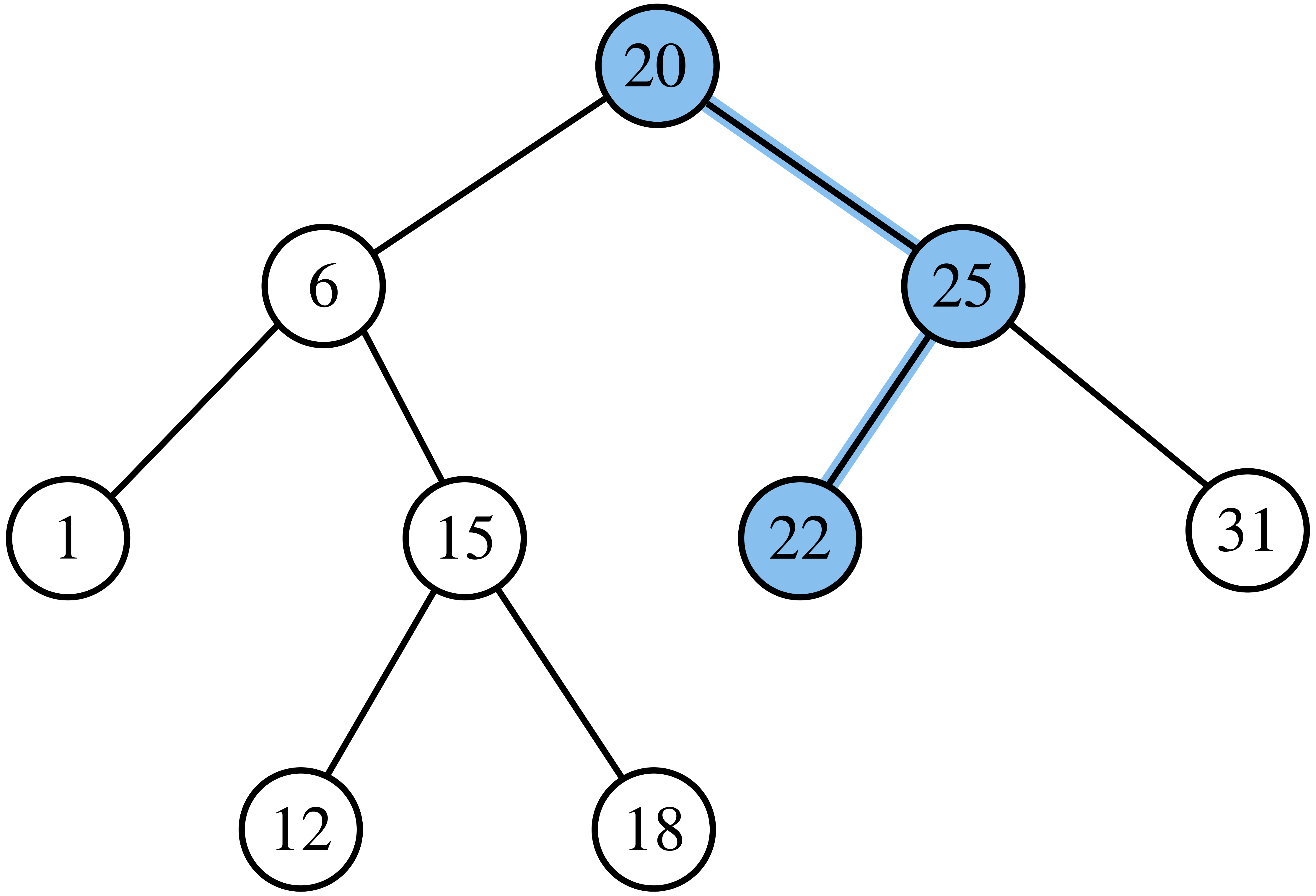

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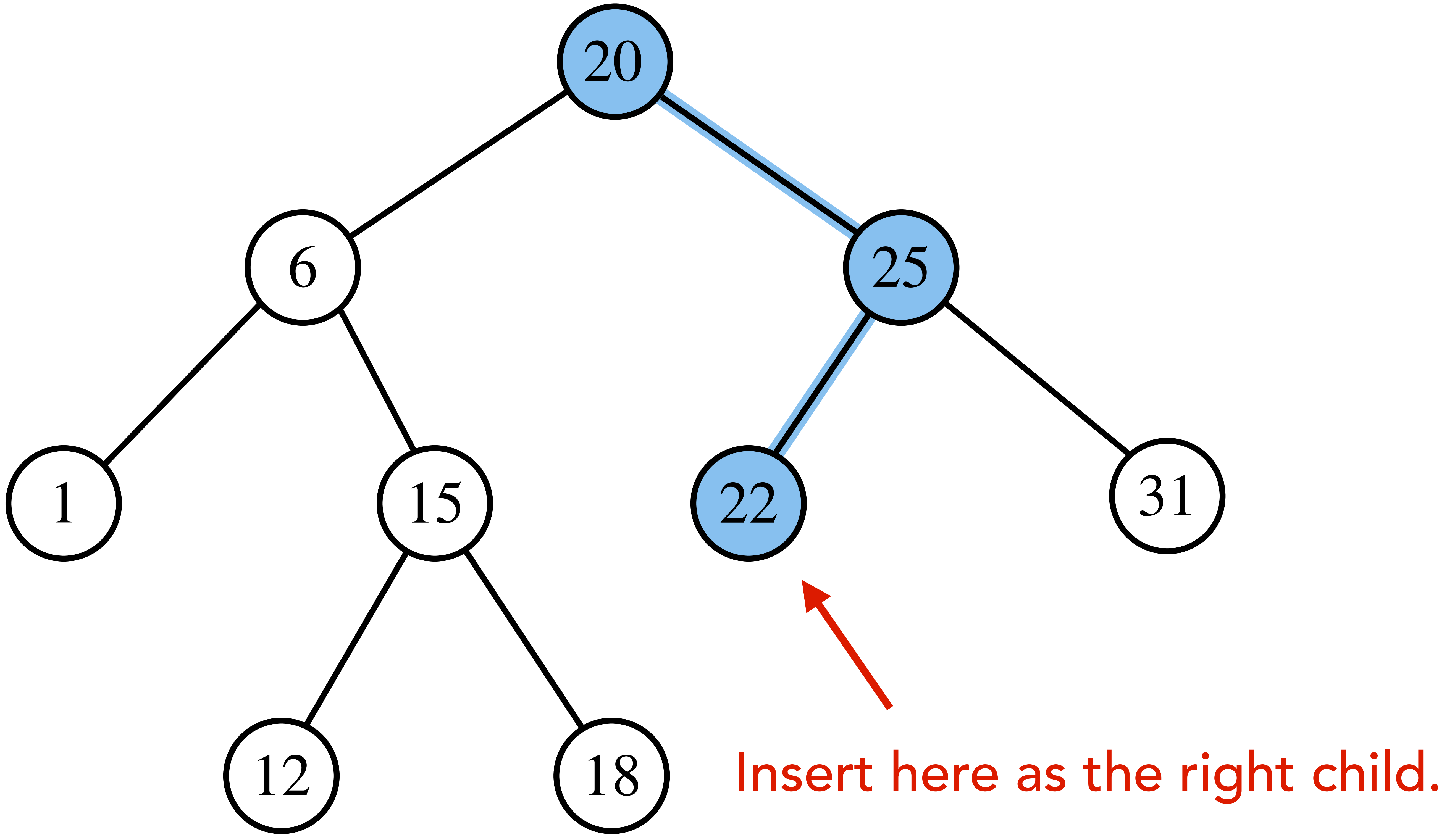

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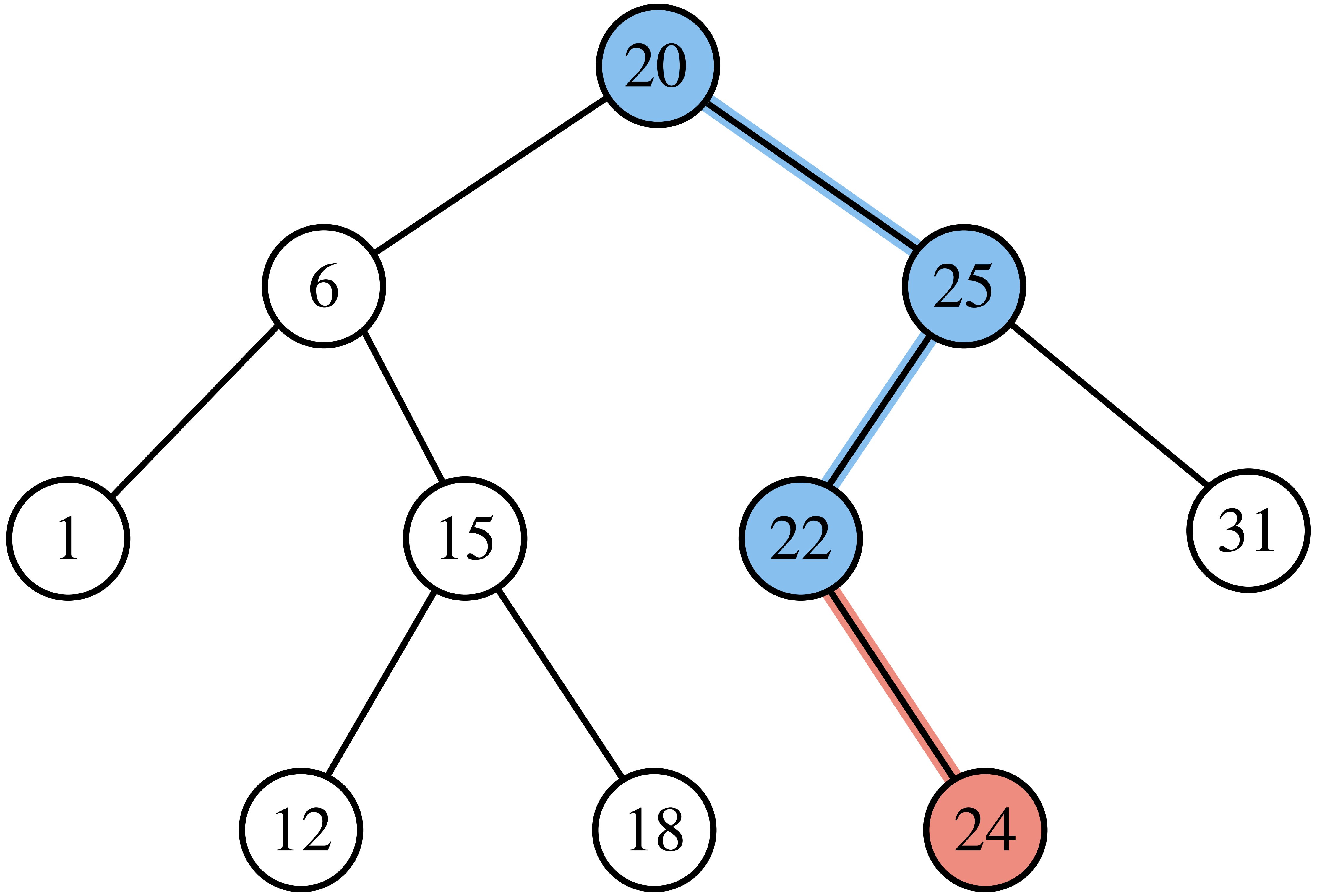

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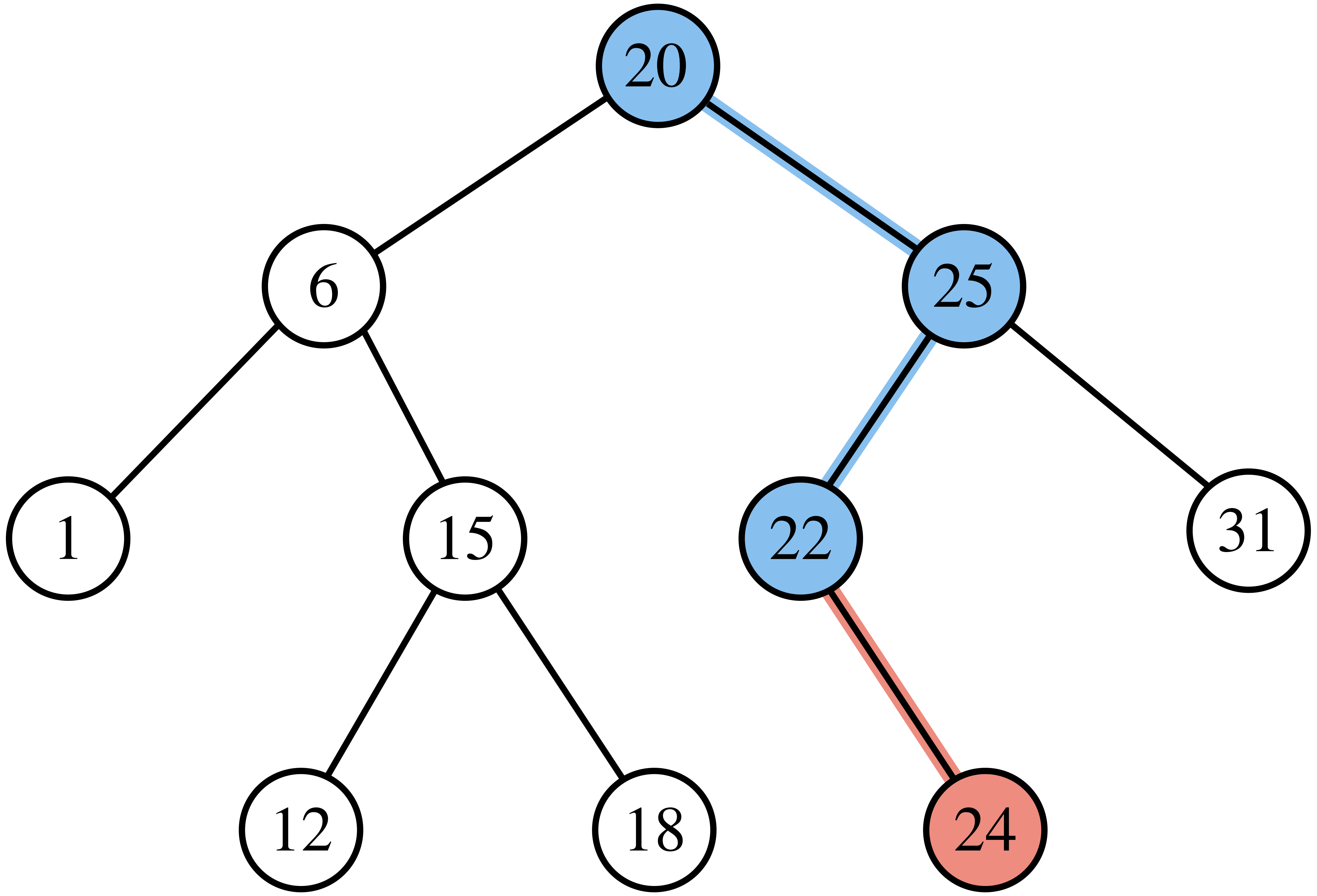

Searching for the correct leaf and insertion takes $\Theta(h)$ time.

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Note: Node z is provided as the input.

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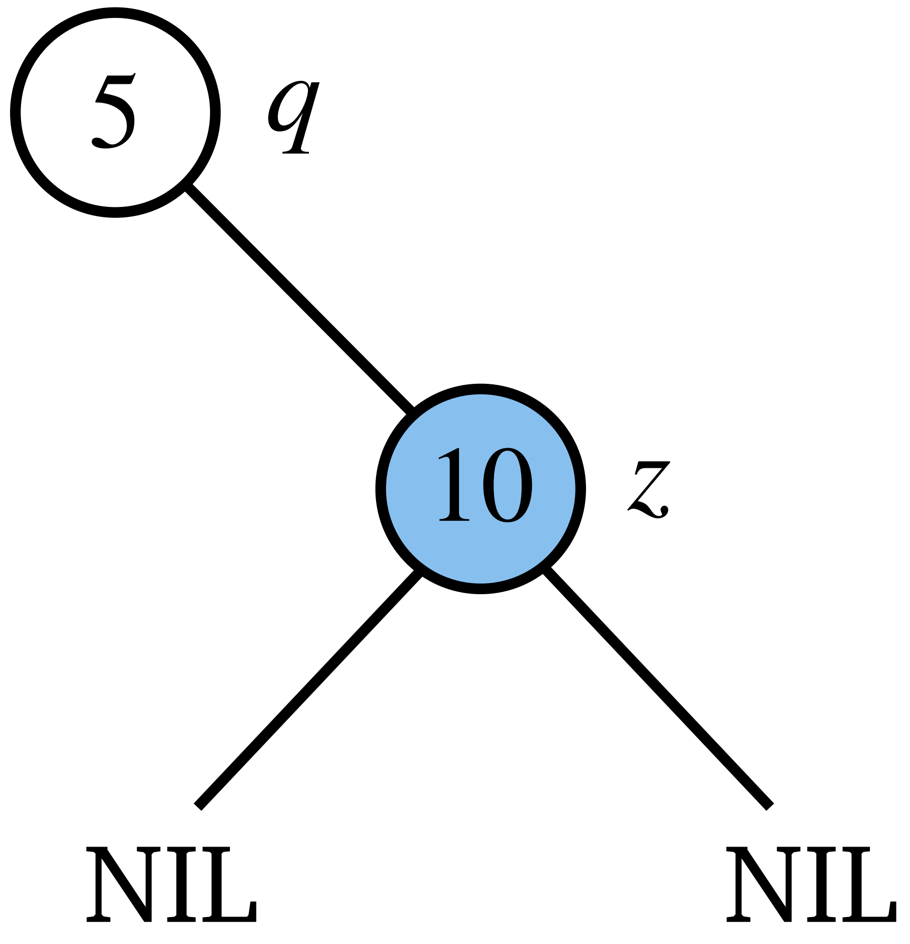

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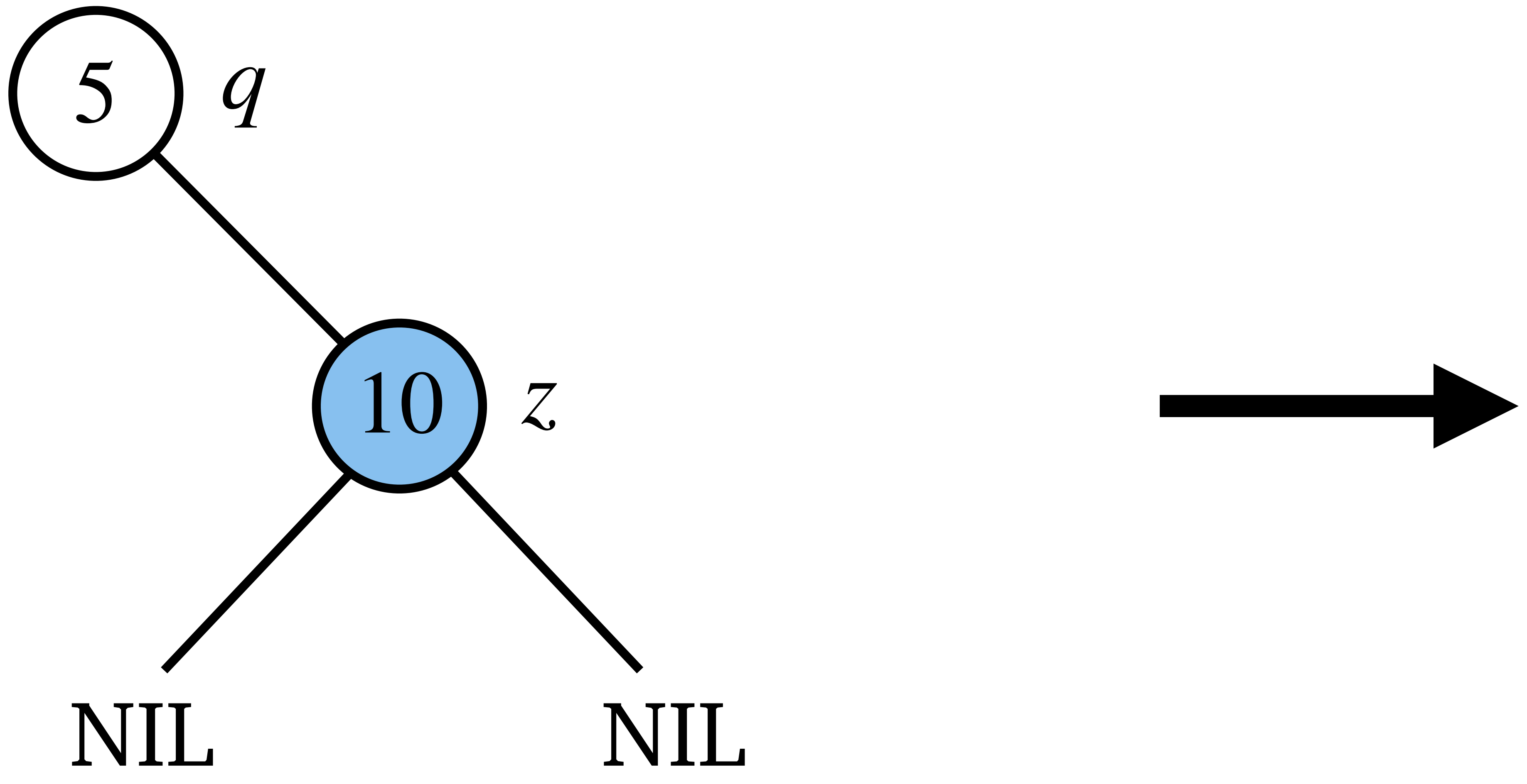

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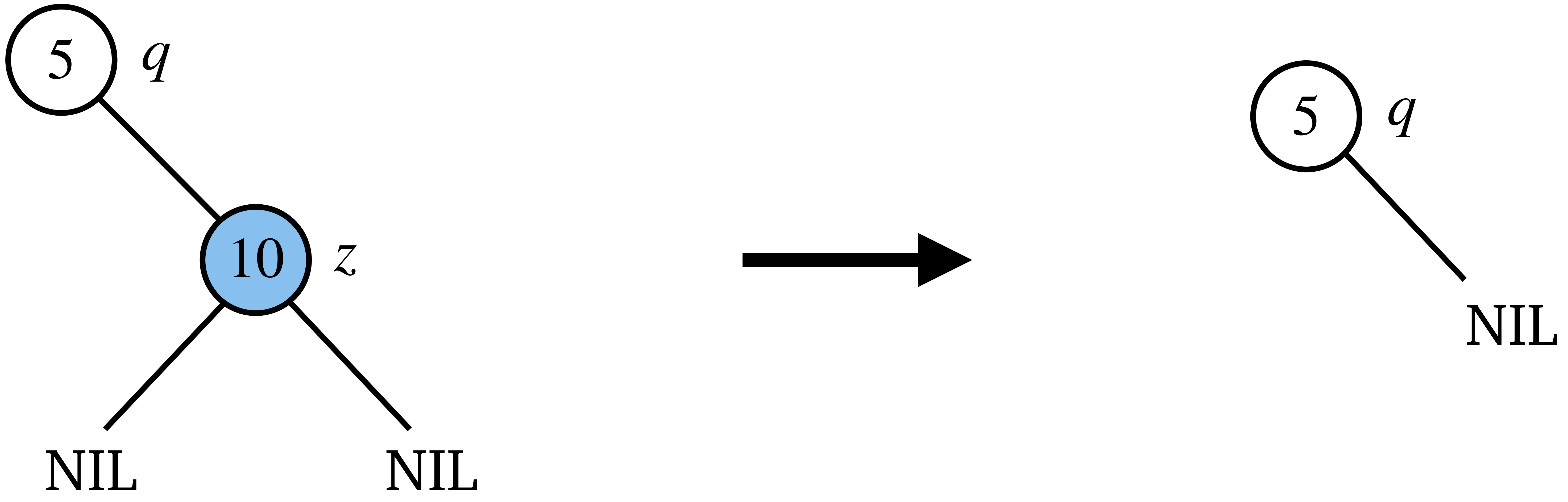

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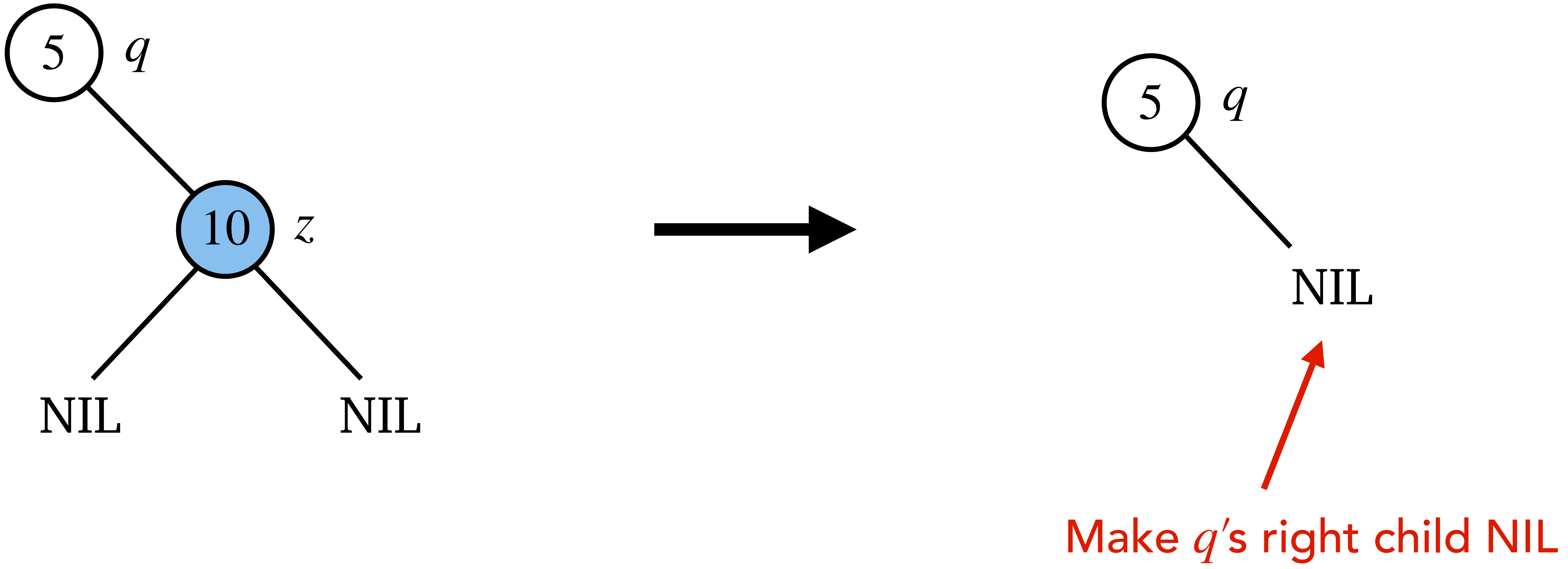

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Case 2: z has one child. (WLOG assume z is a right child.)

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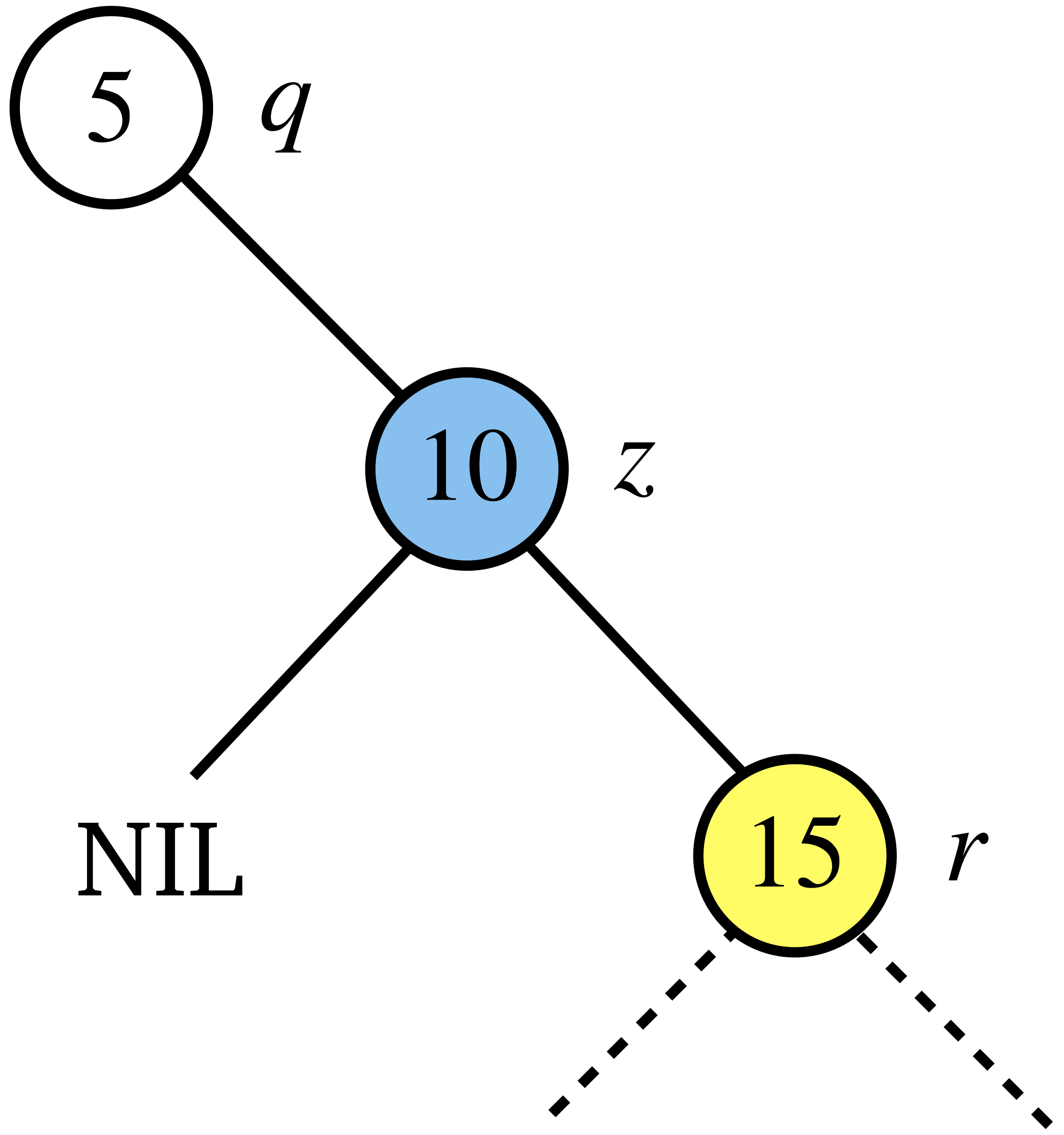

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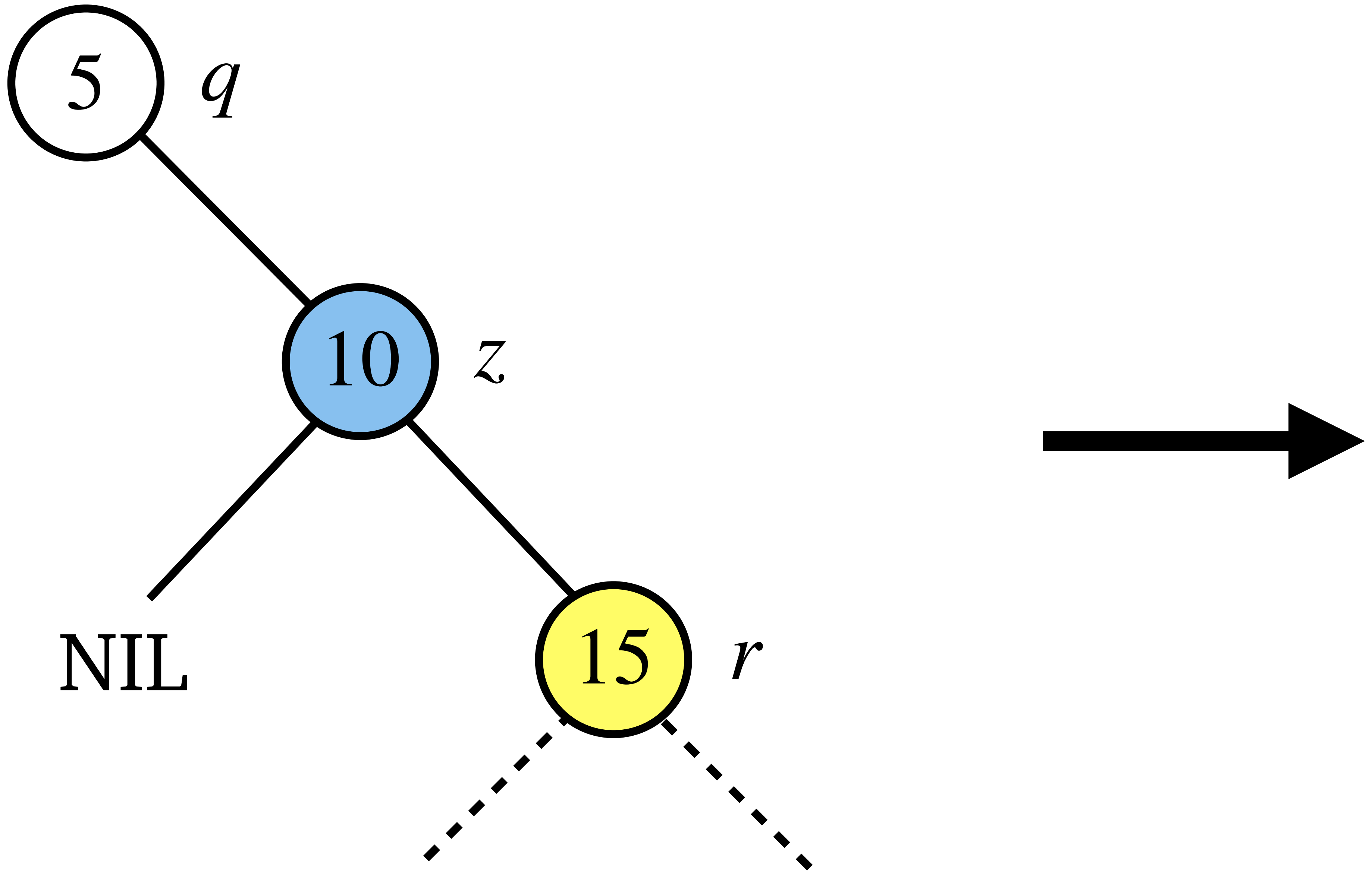

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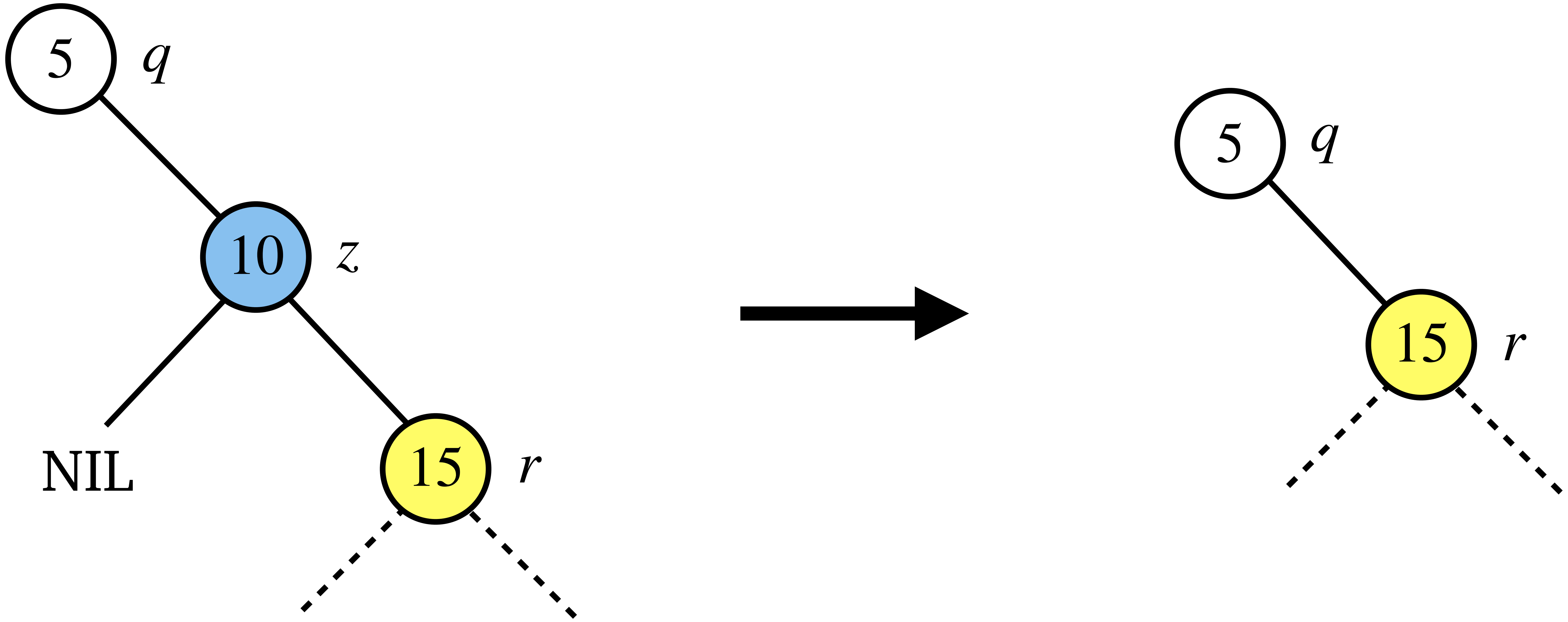

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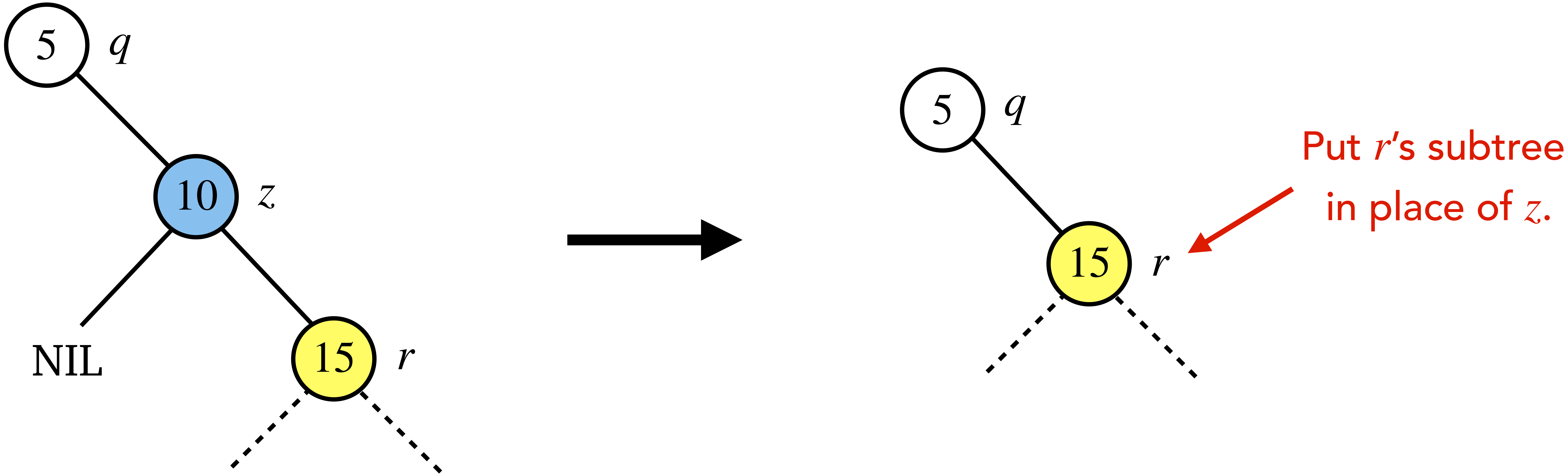

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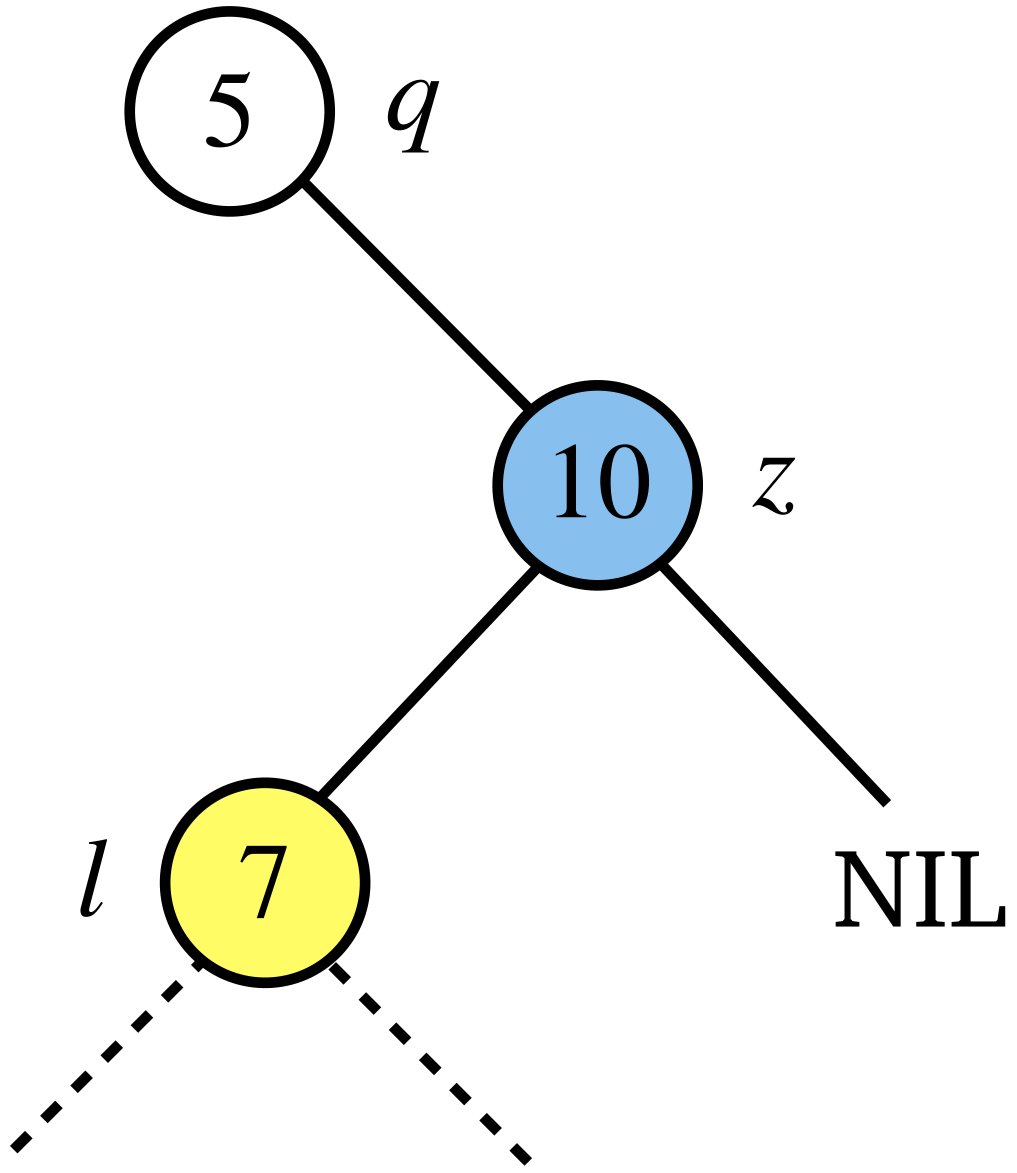

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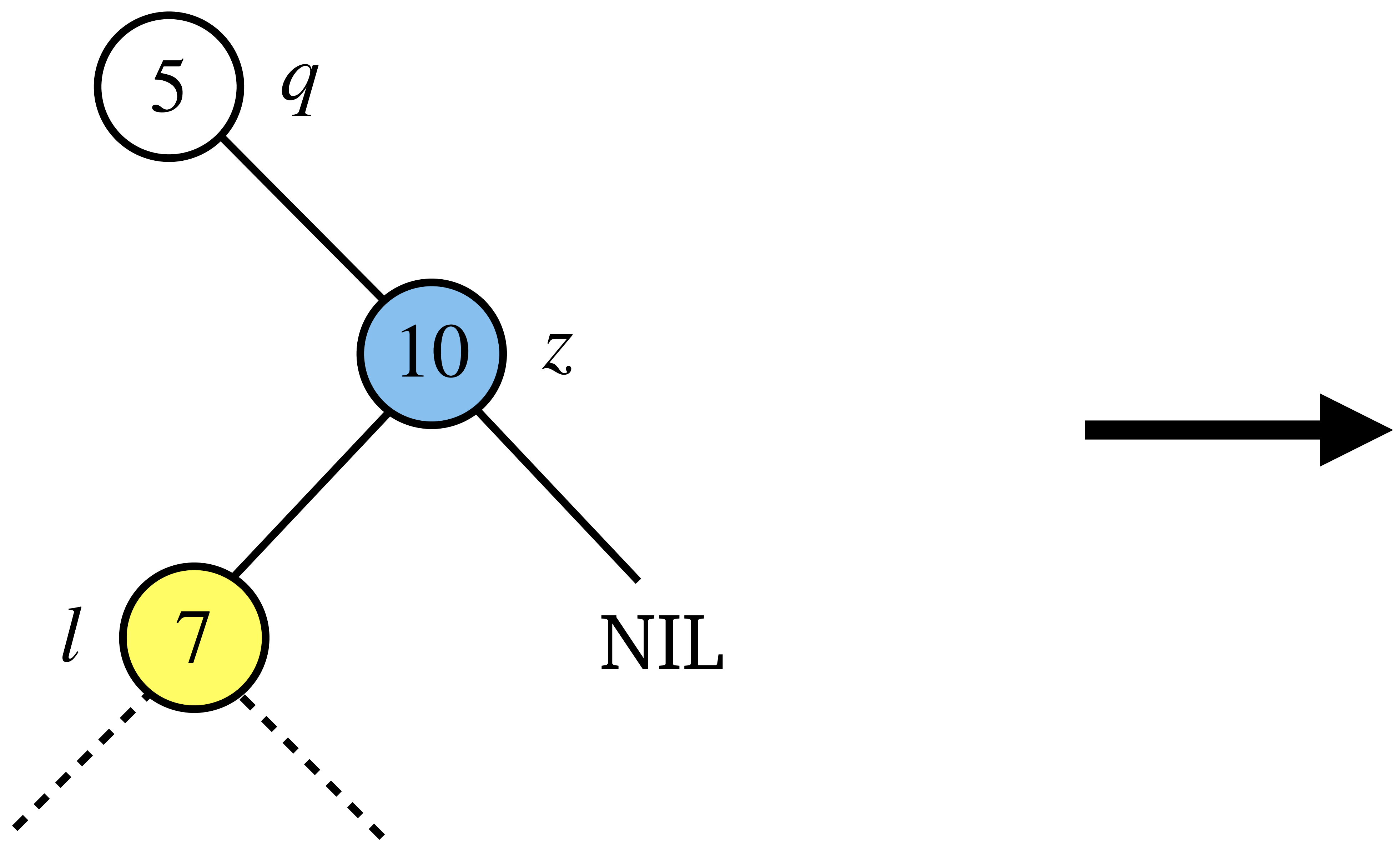

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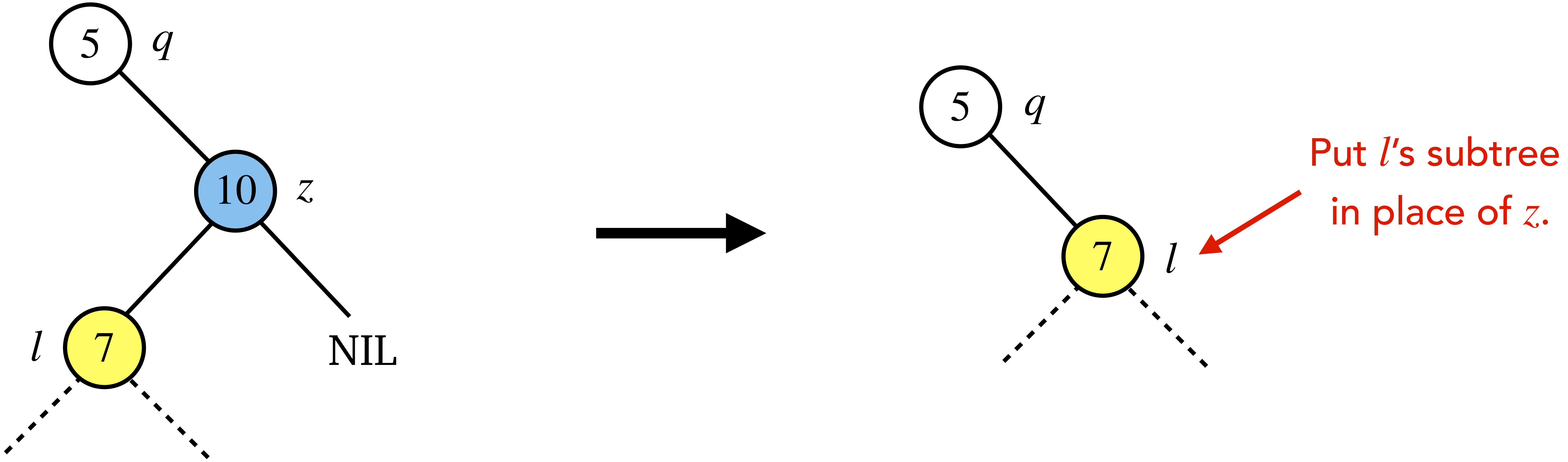

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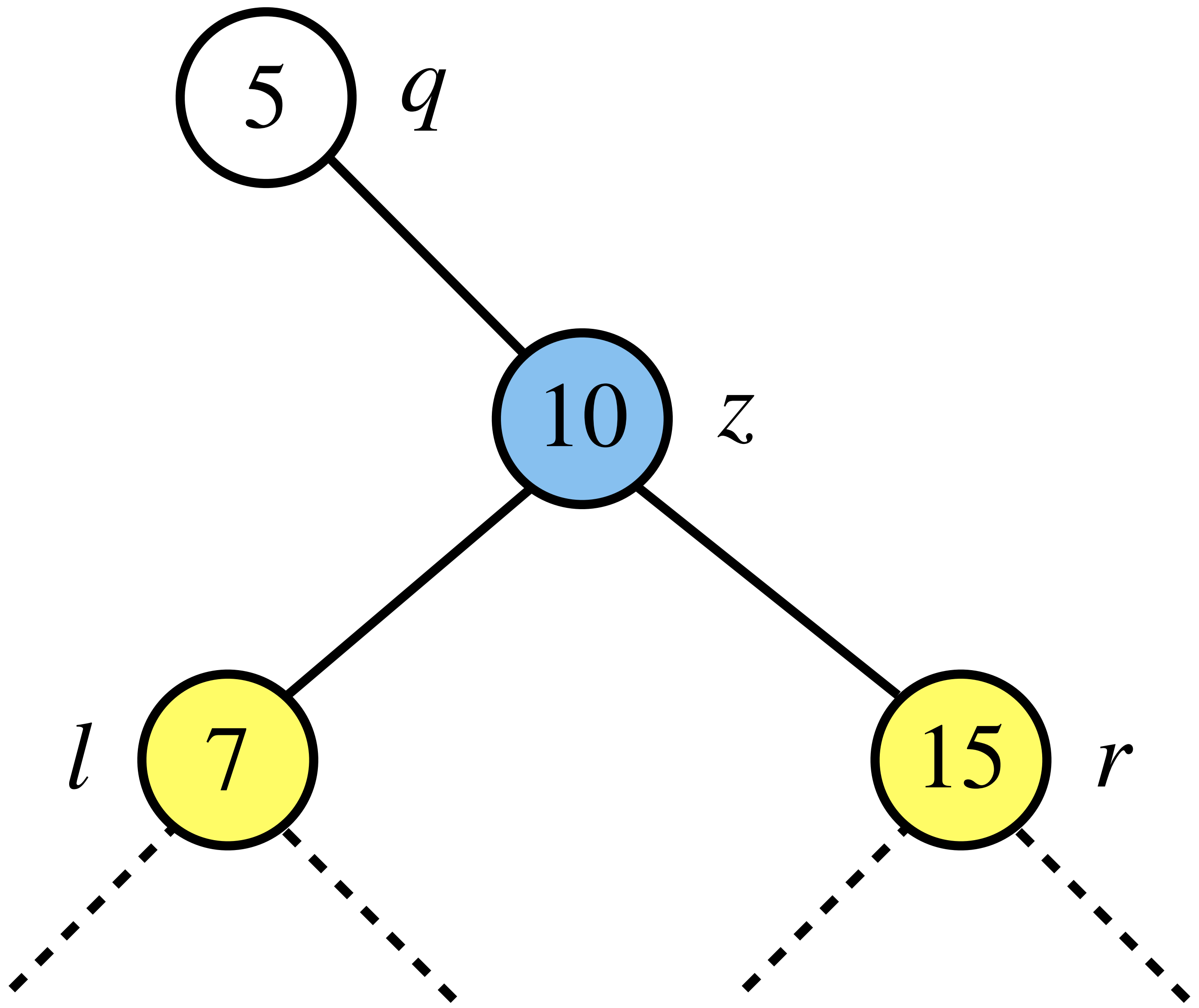

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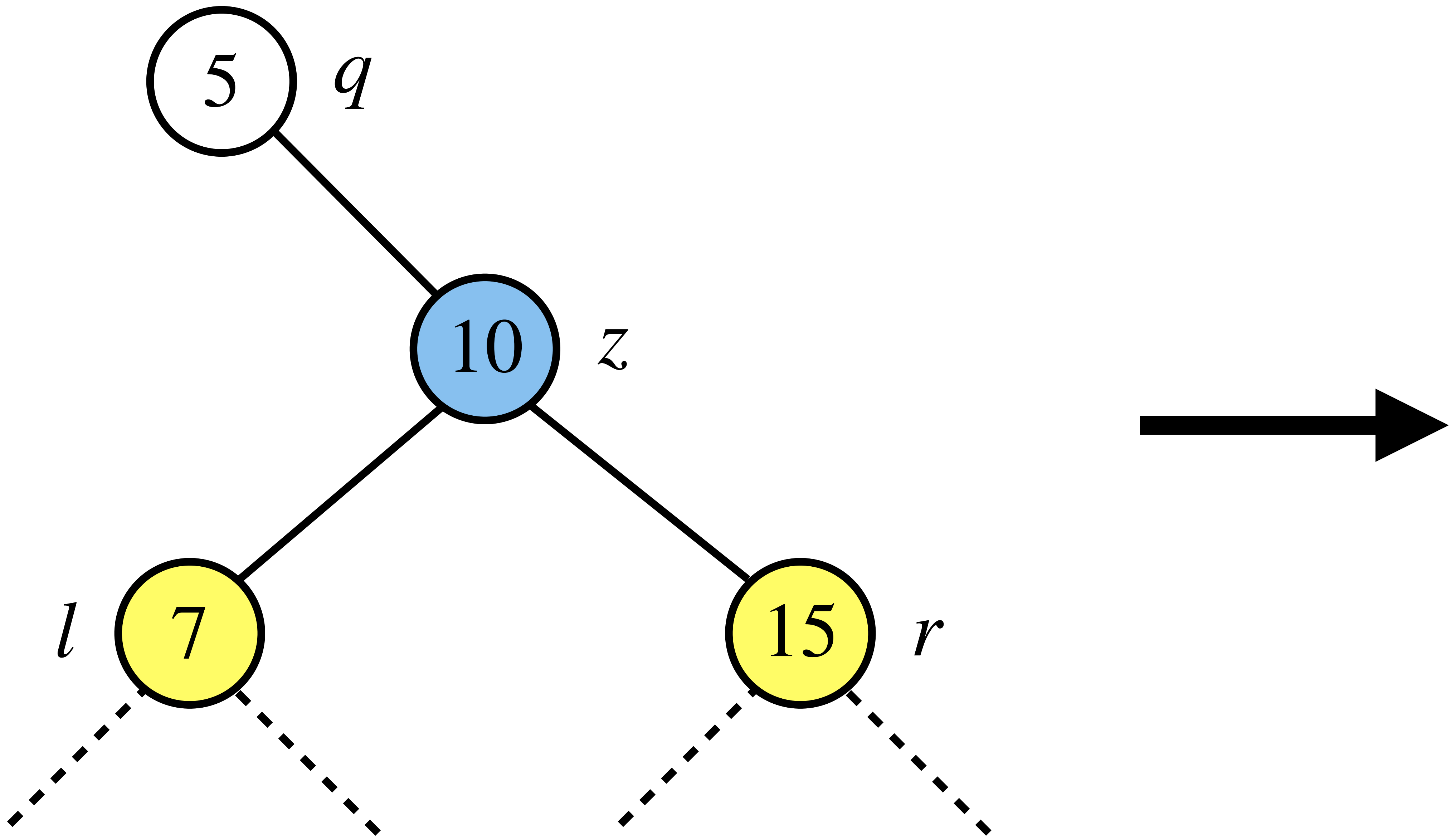

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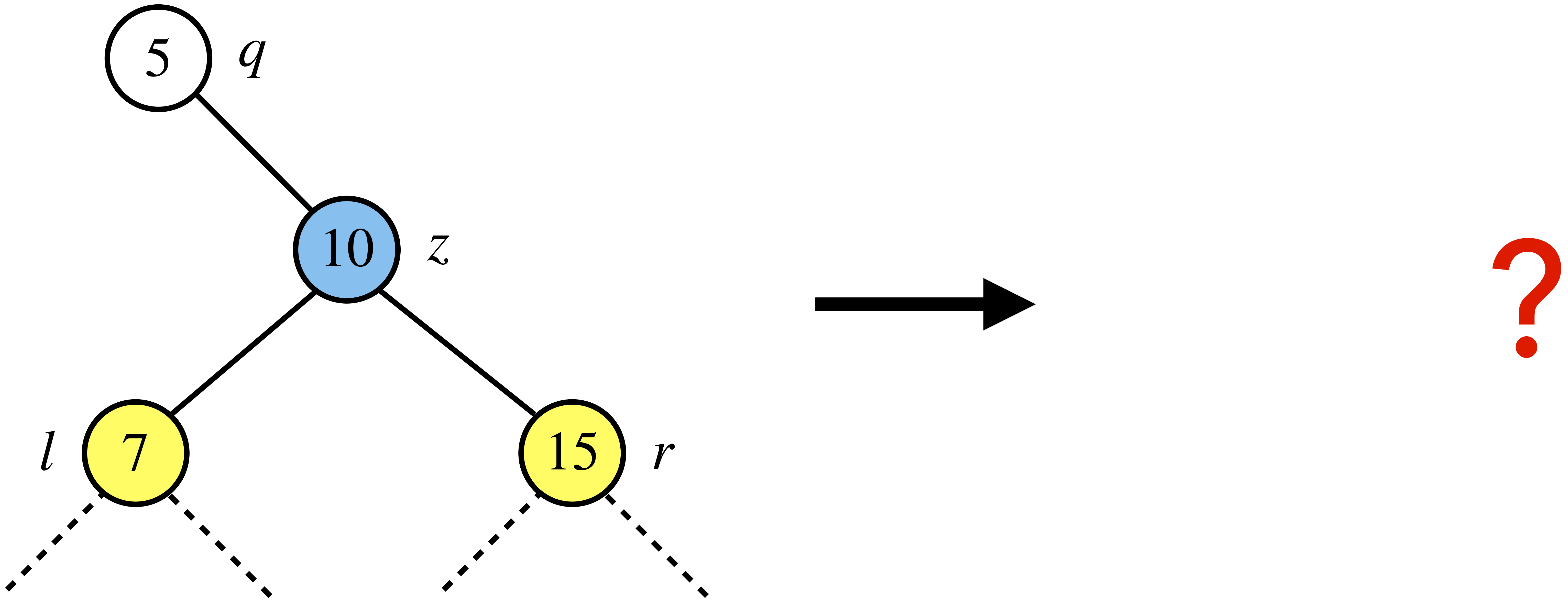

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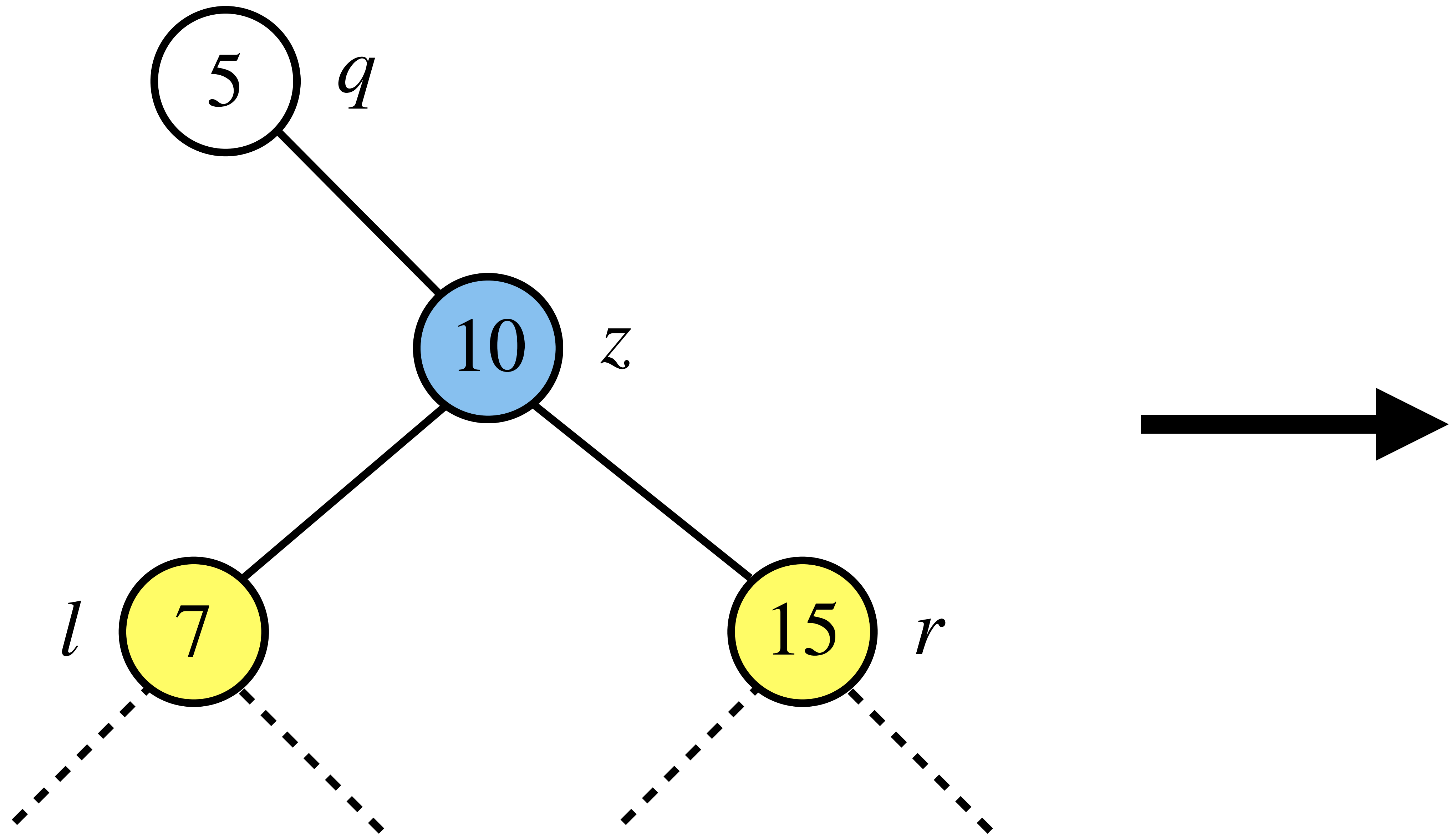

Two sub-cases:

- r has no left child.
- r has a left child.

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Case 3a: z has two children where its right child has no left child.

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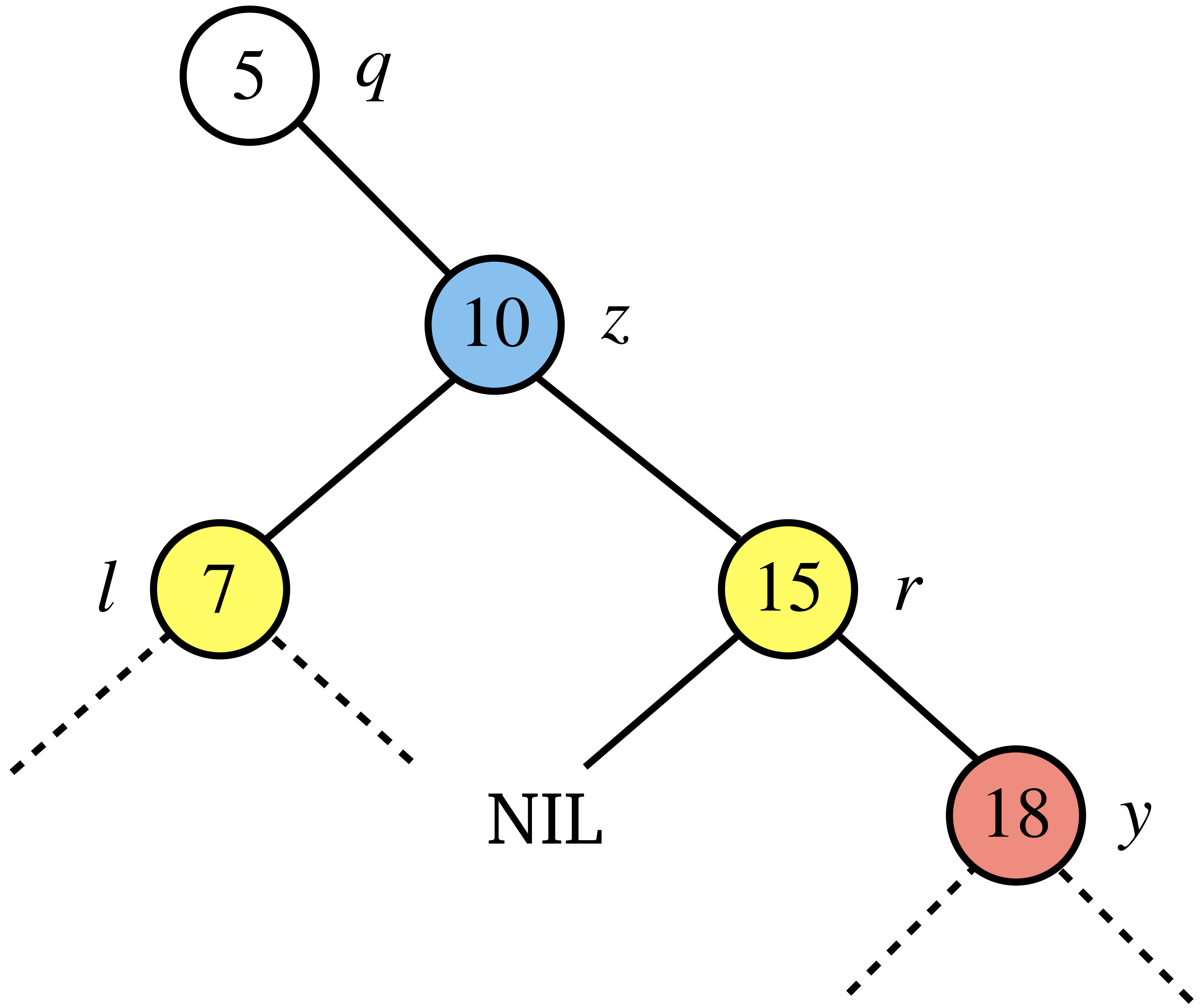
Case 3**a**: z has two children where its right child has no left child.



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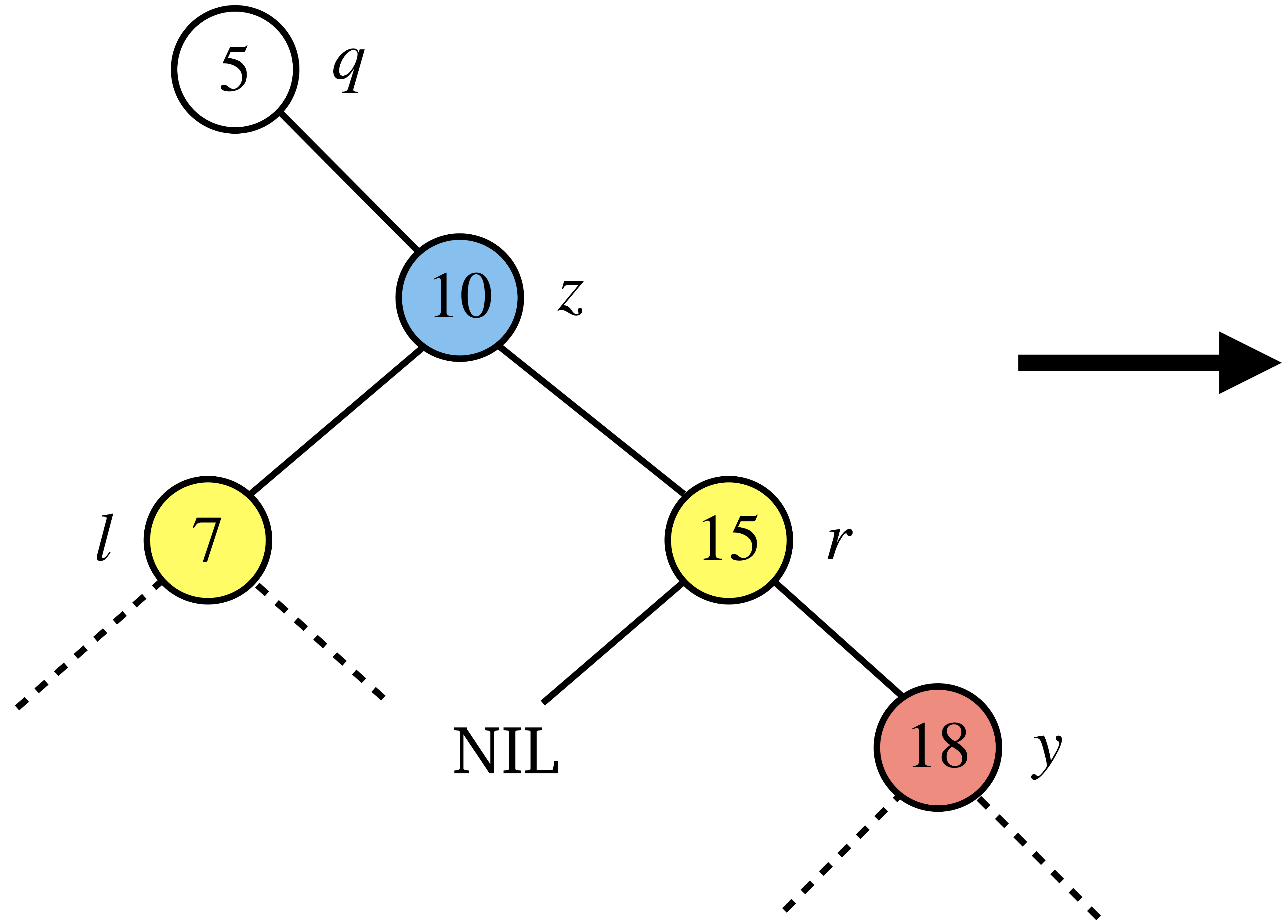
Case 3**a**: z has two children where its right child has no left child.



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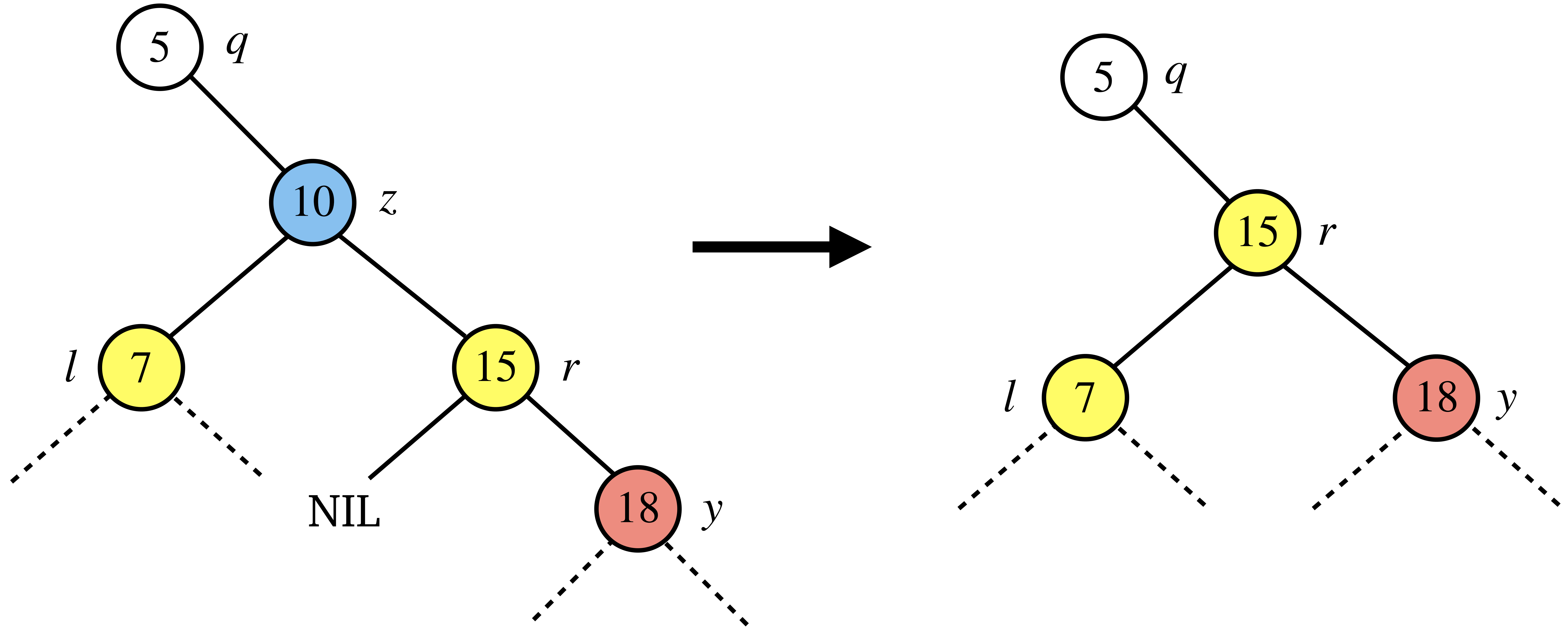

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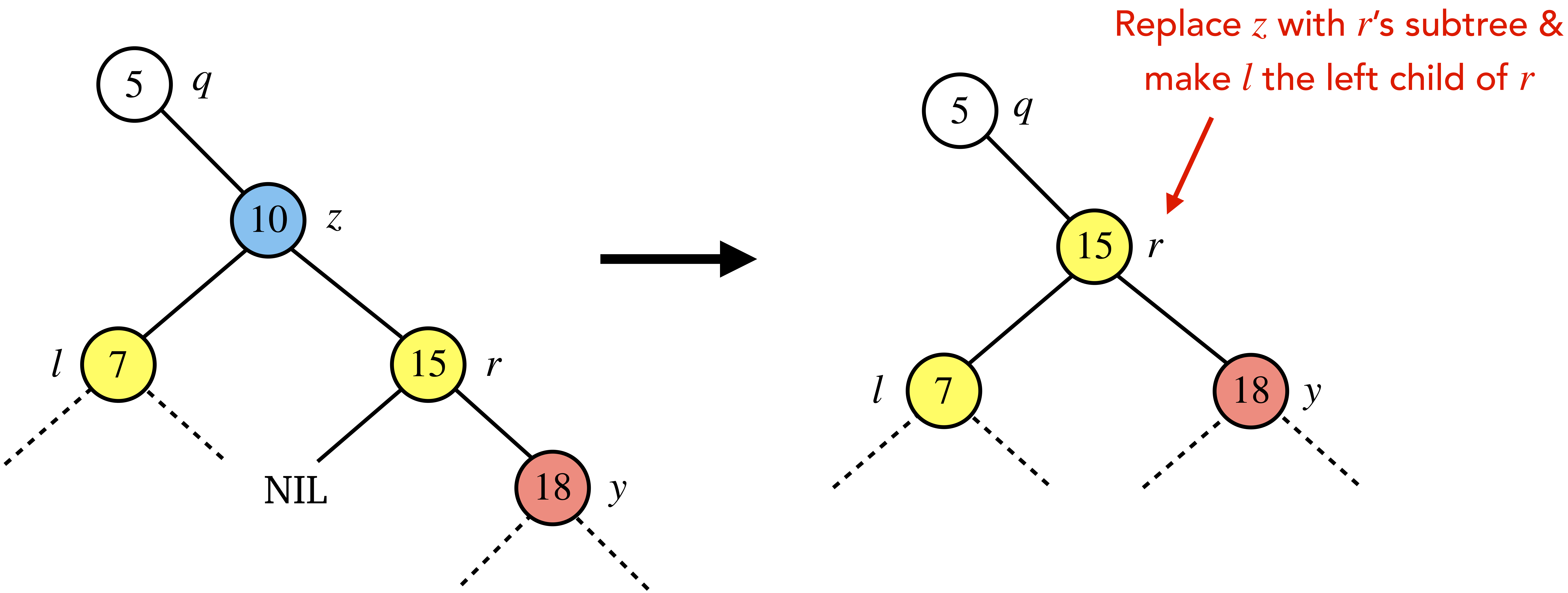

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Case 3b: z has two children where its right child has a left child.

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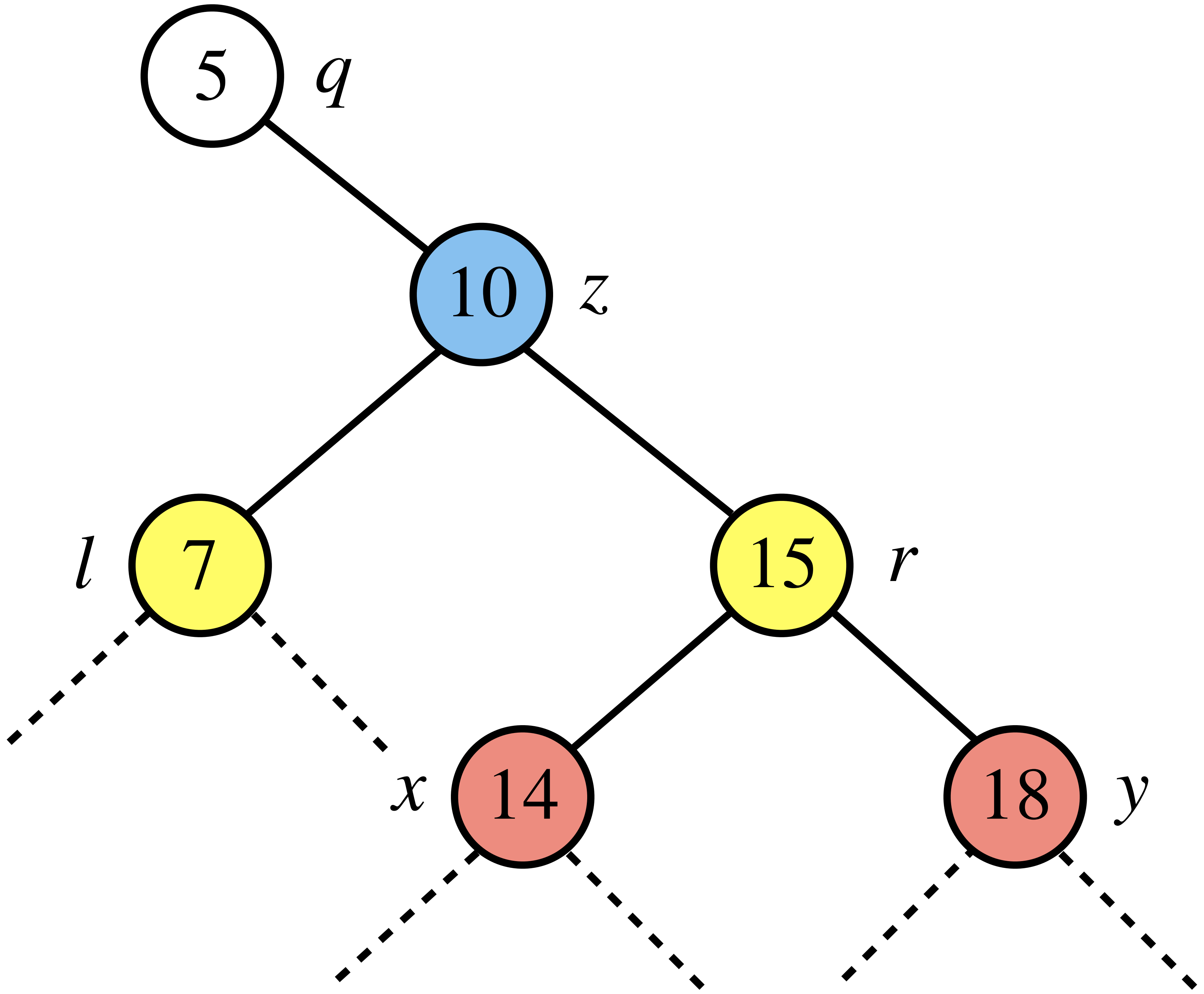
Case 3**b**: z has two children where its right child has a left child.



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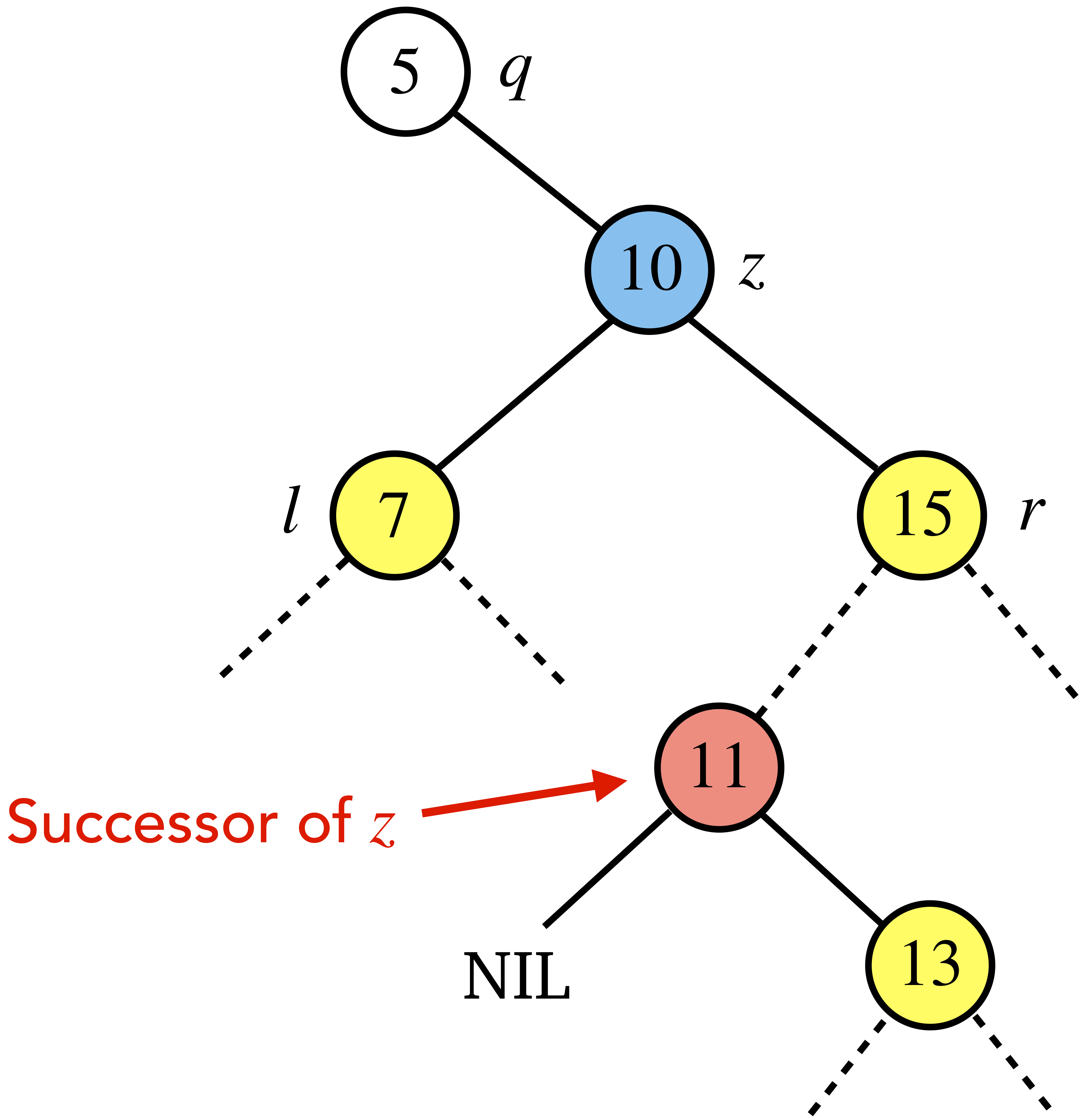
Case 3**b**: z has two children where its right child has a left child.



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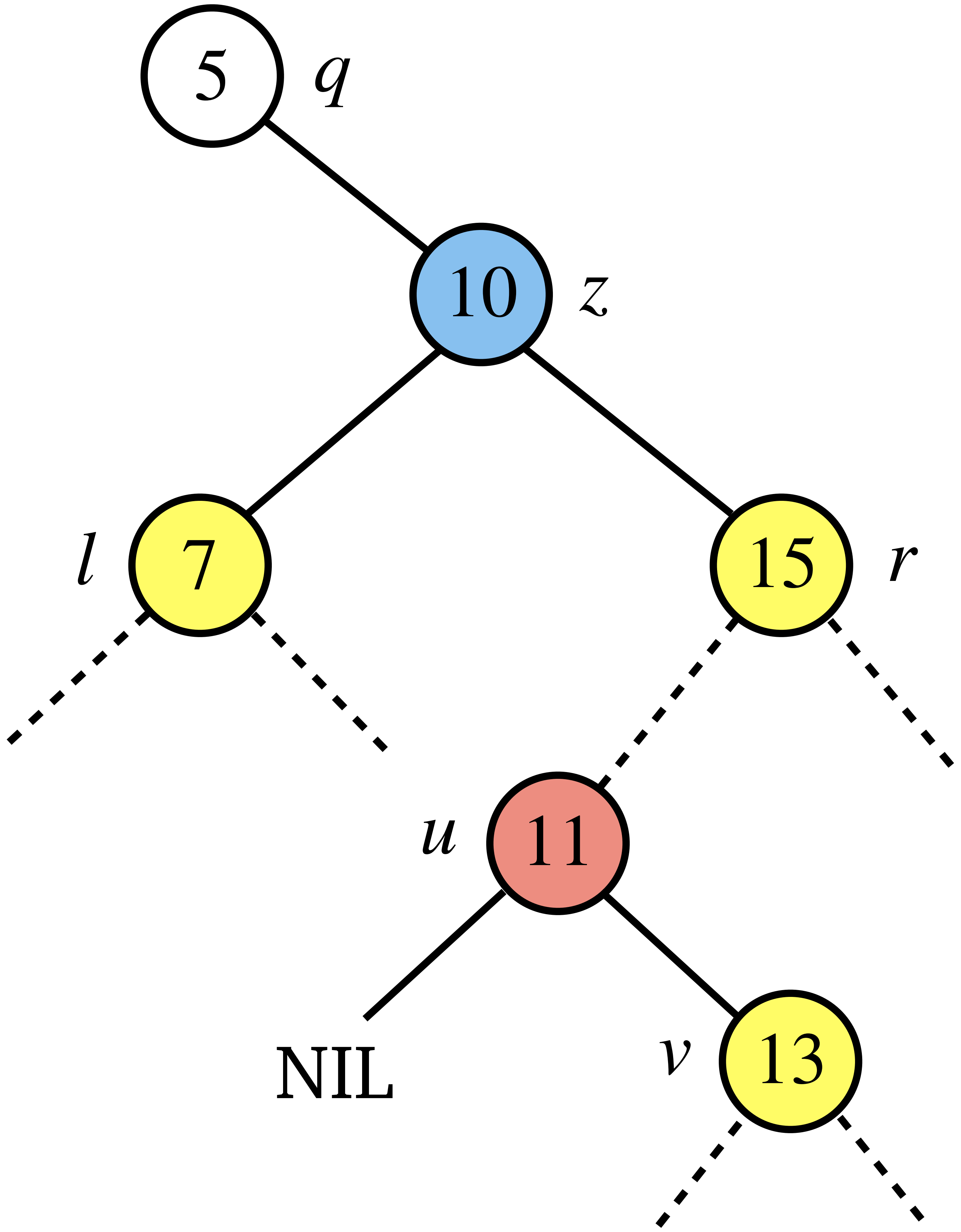

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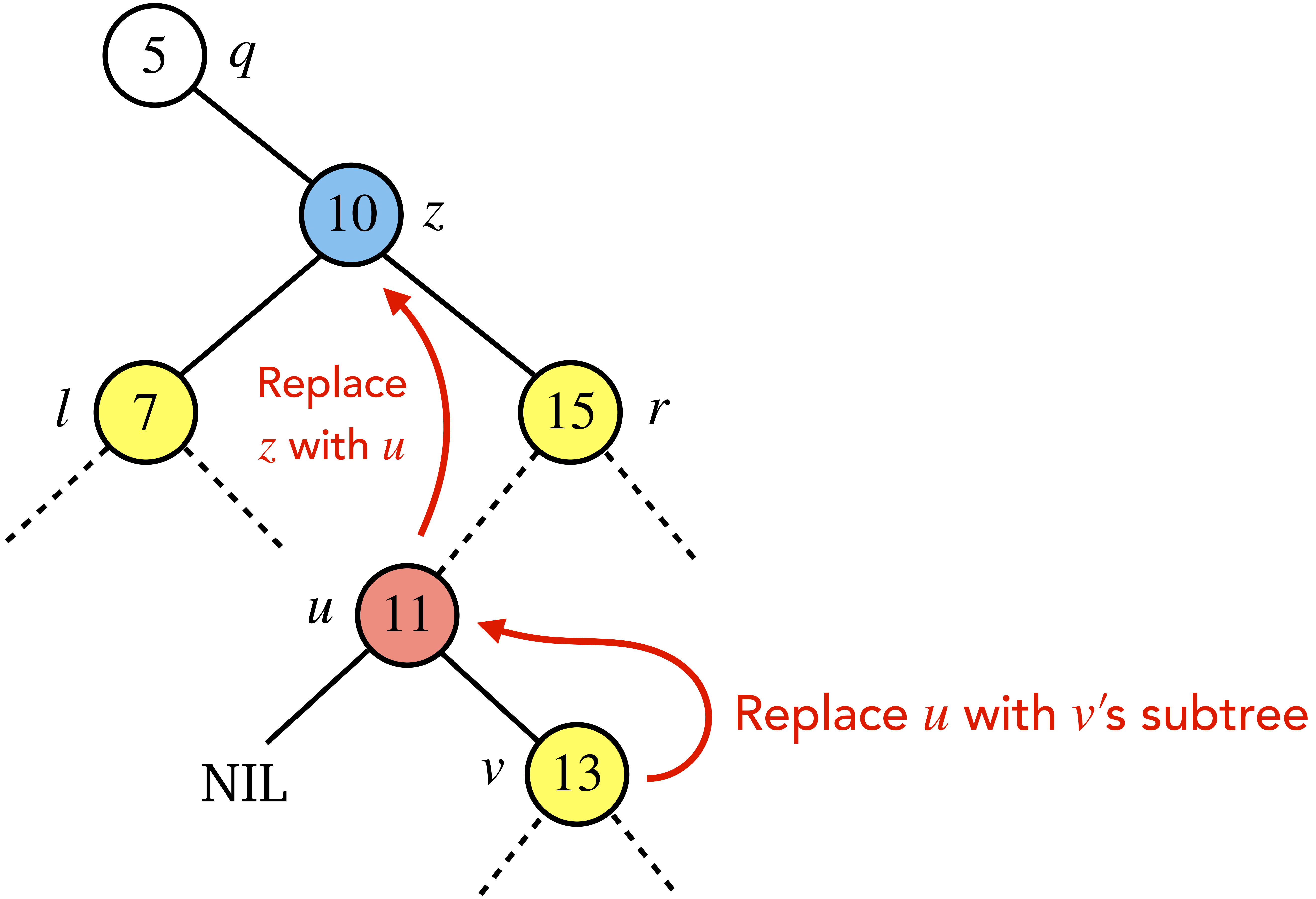

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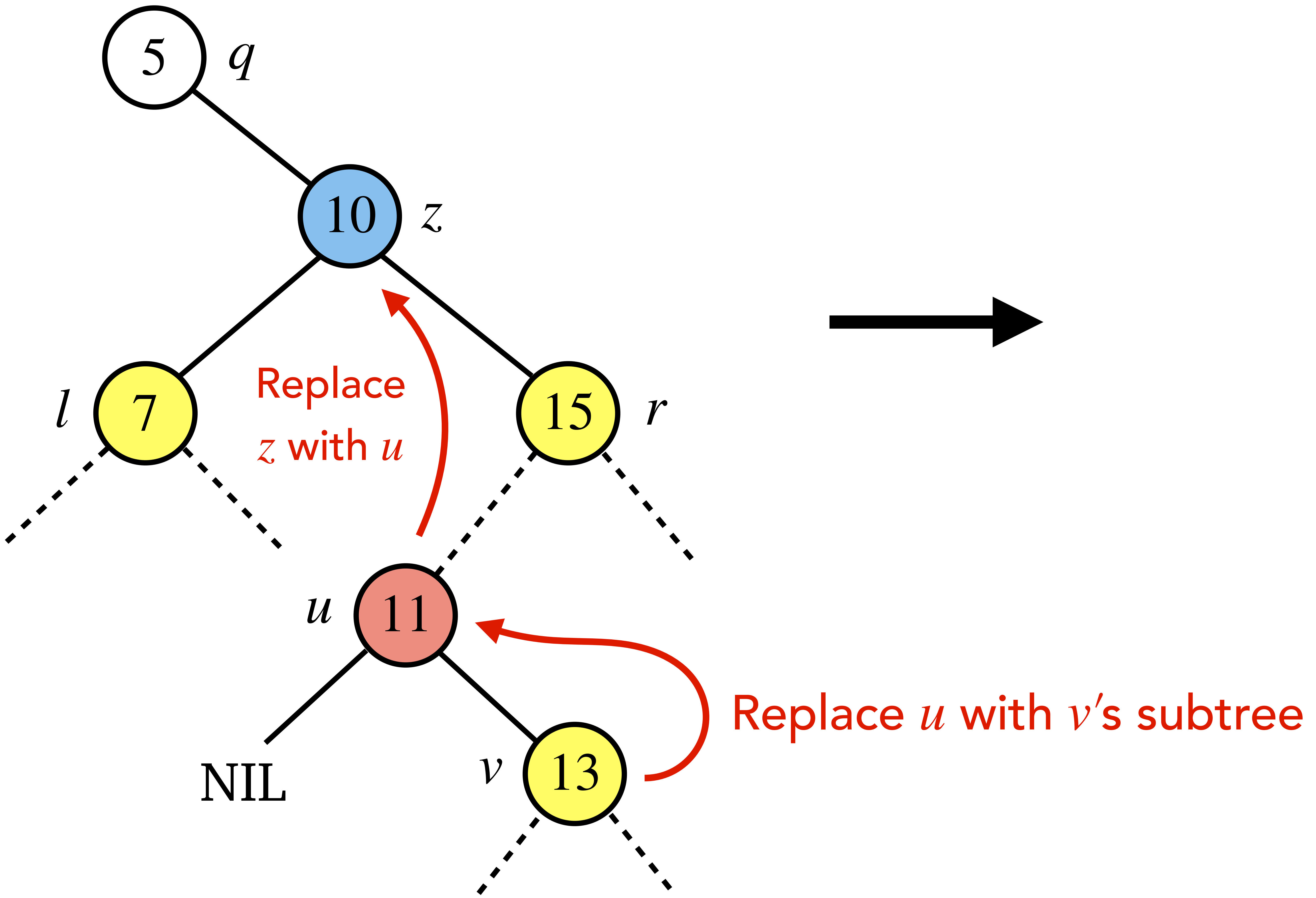

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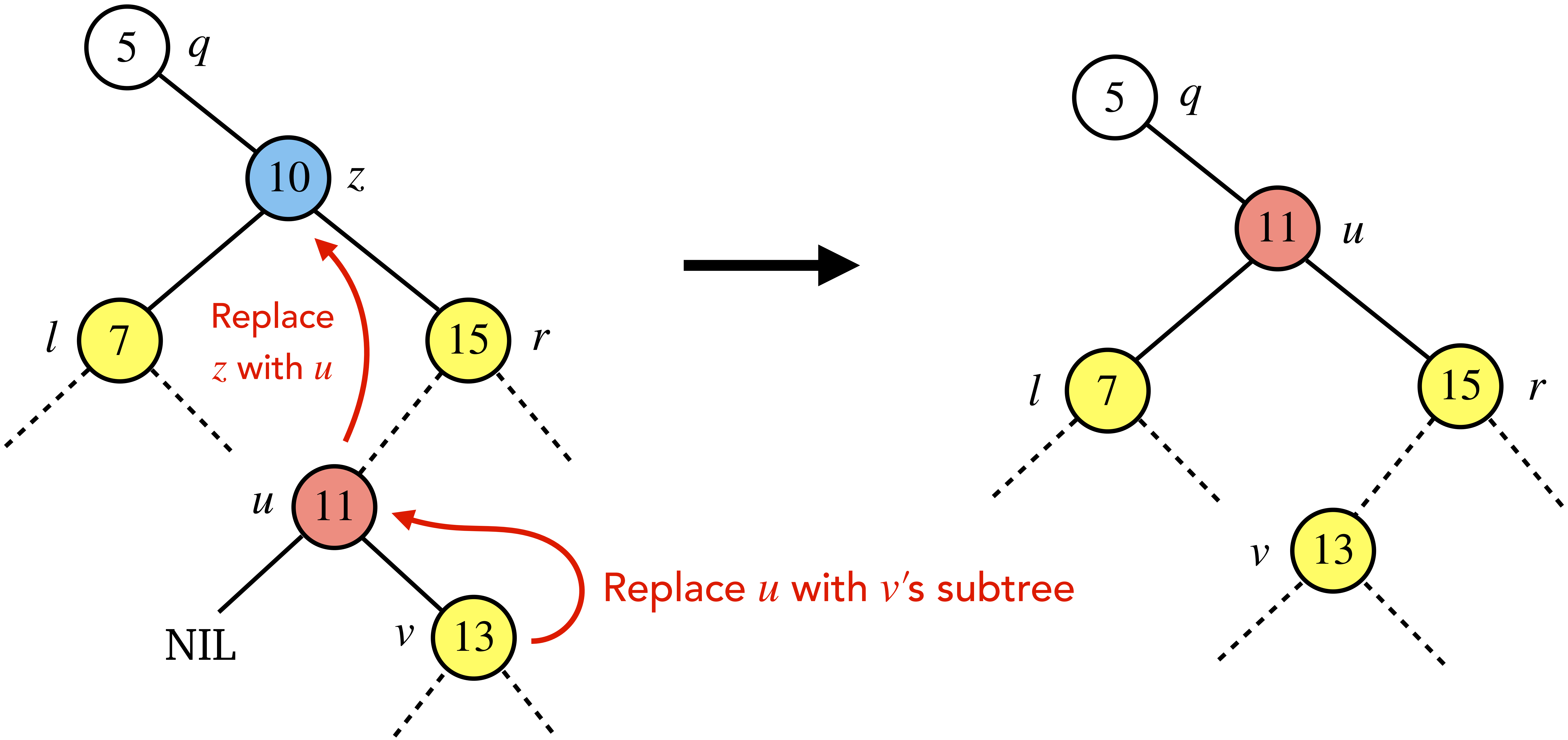

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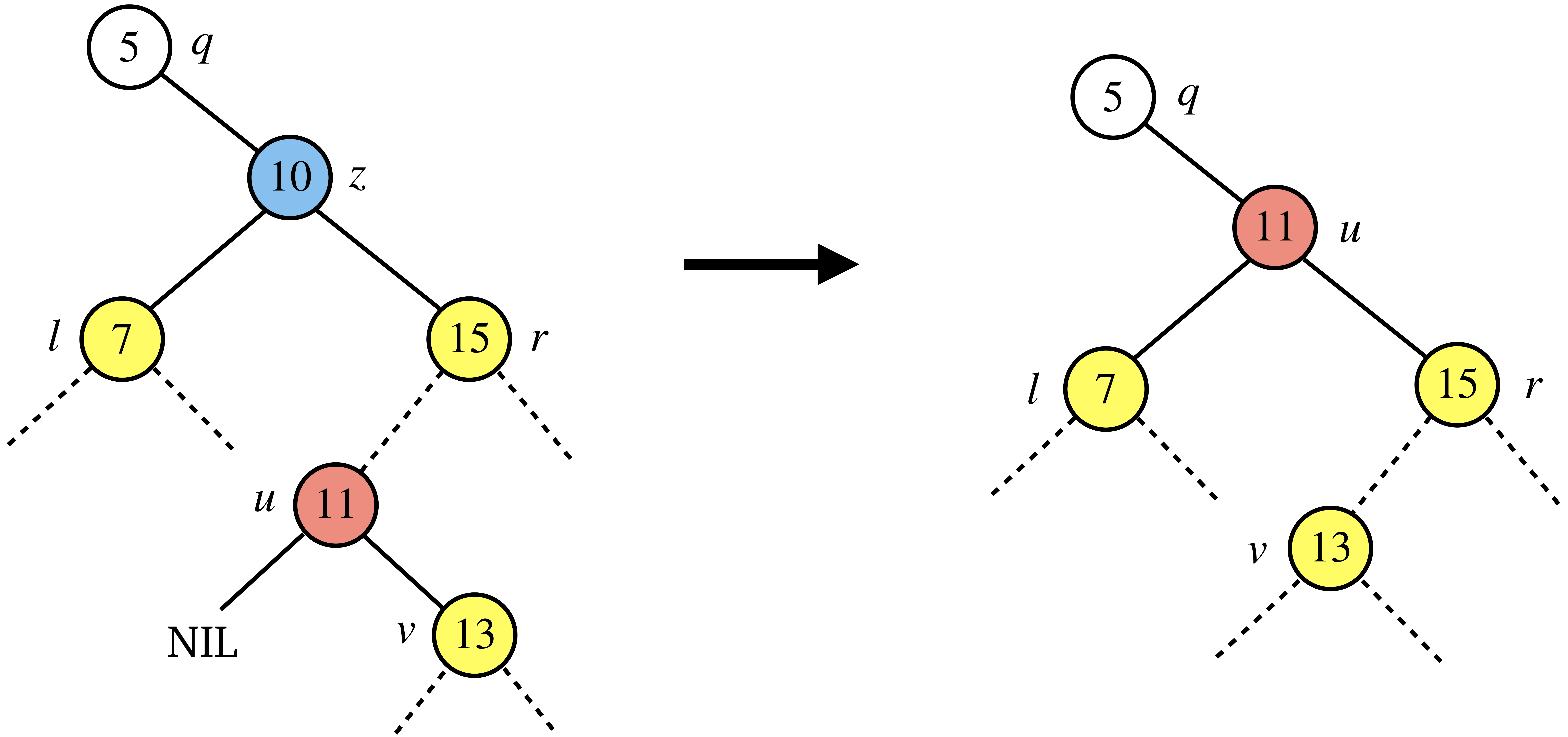

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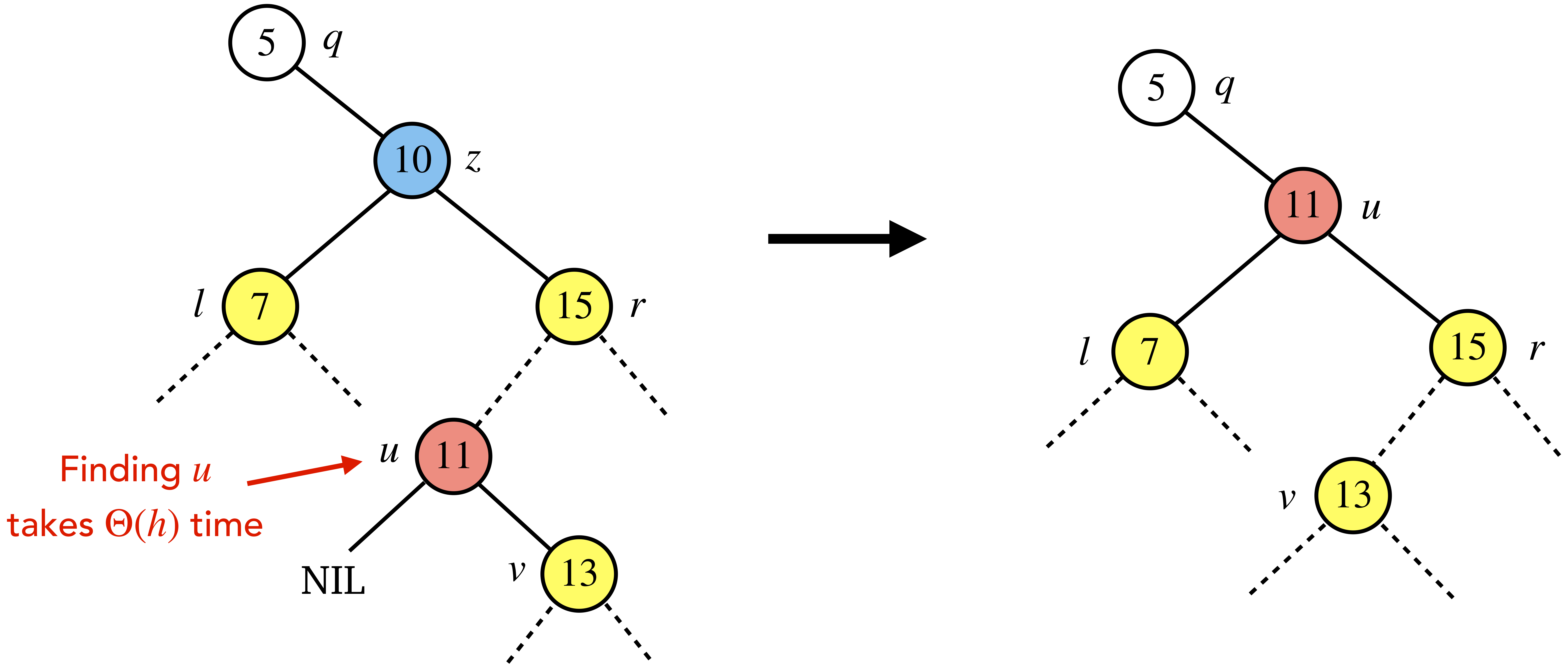

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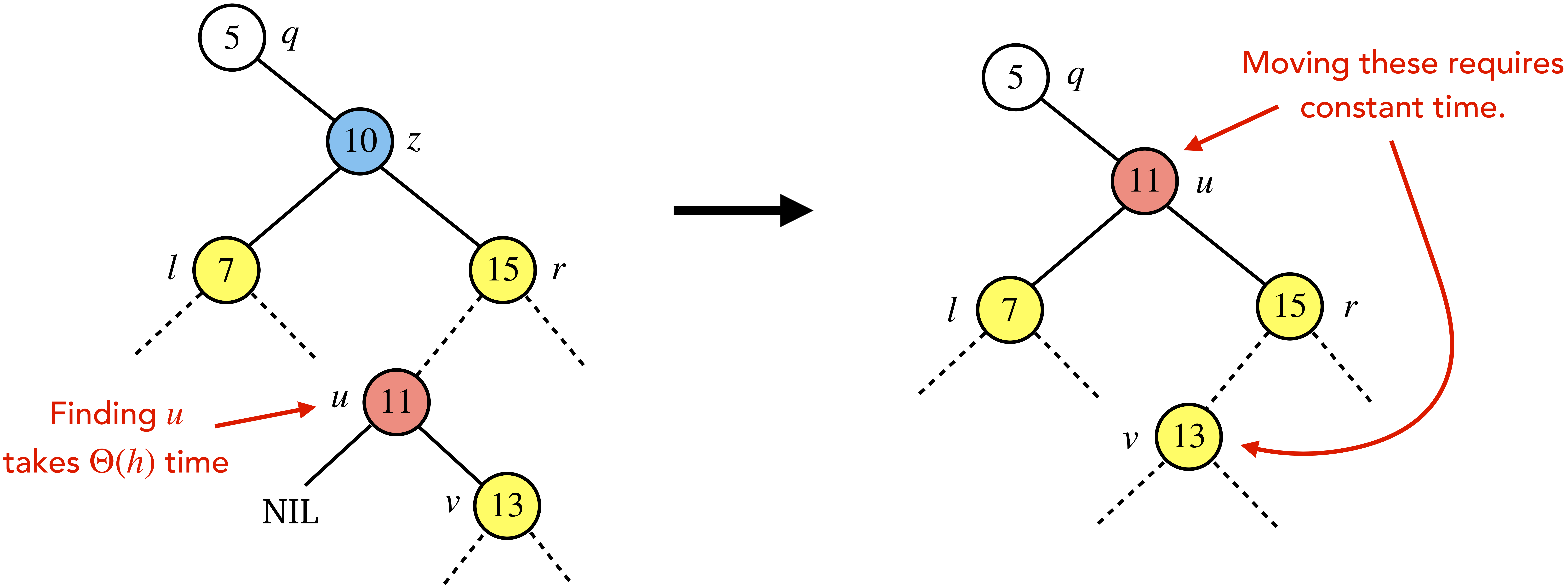

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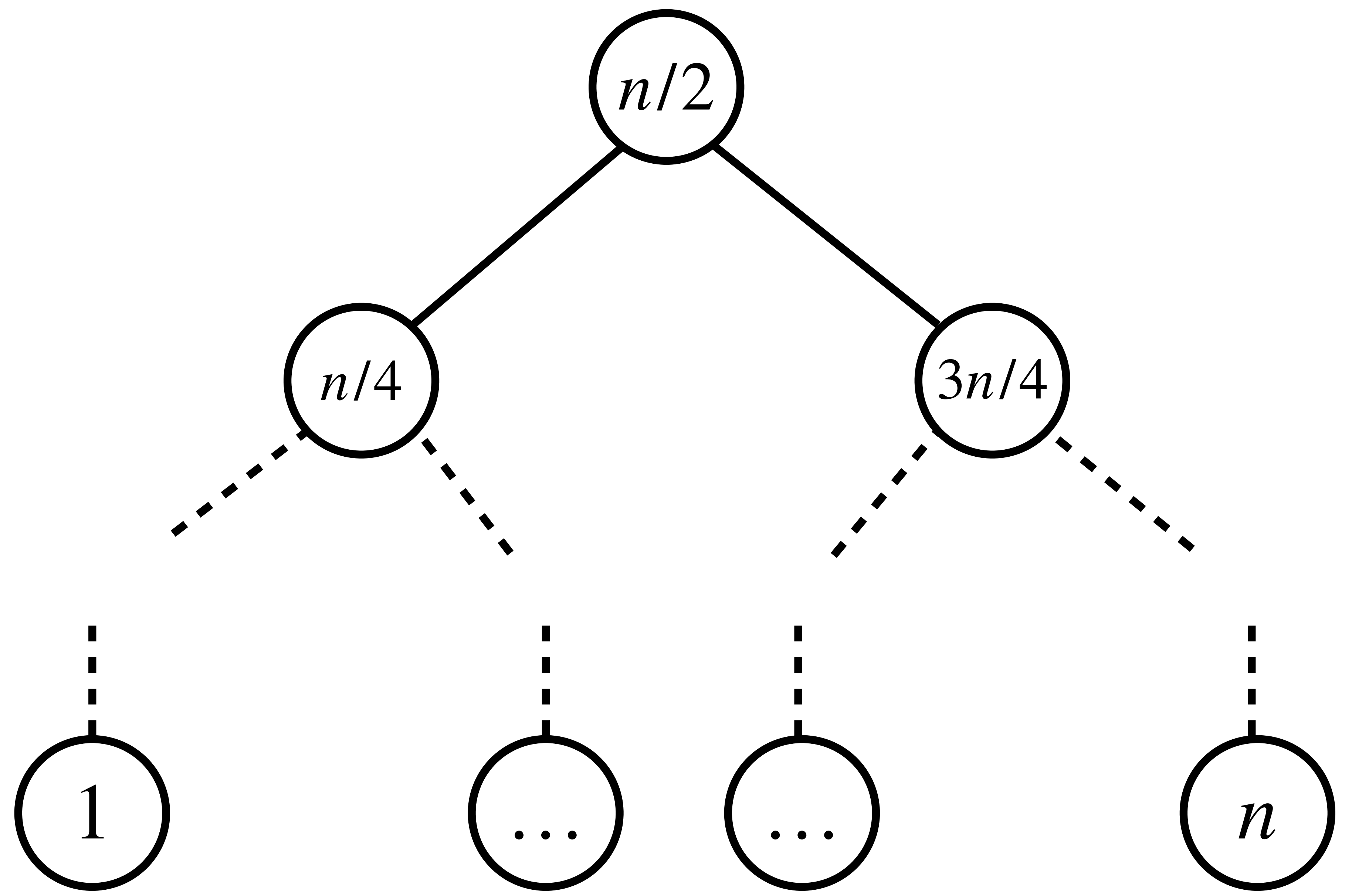

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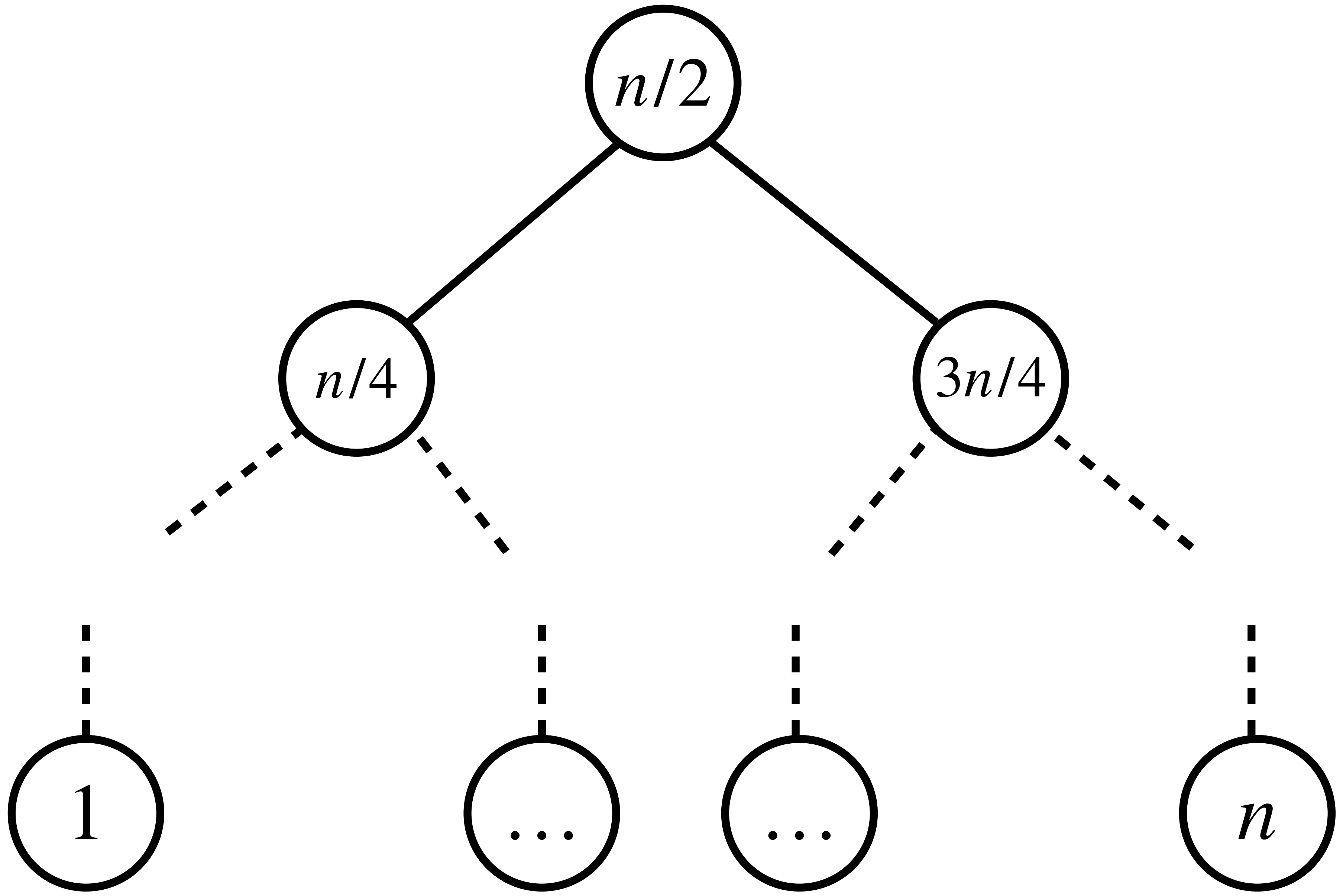

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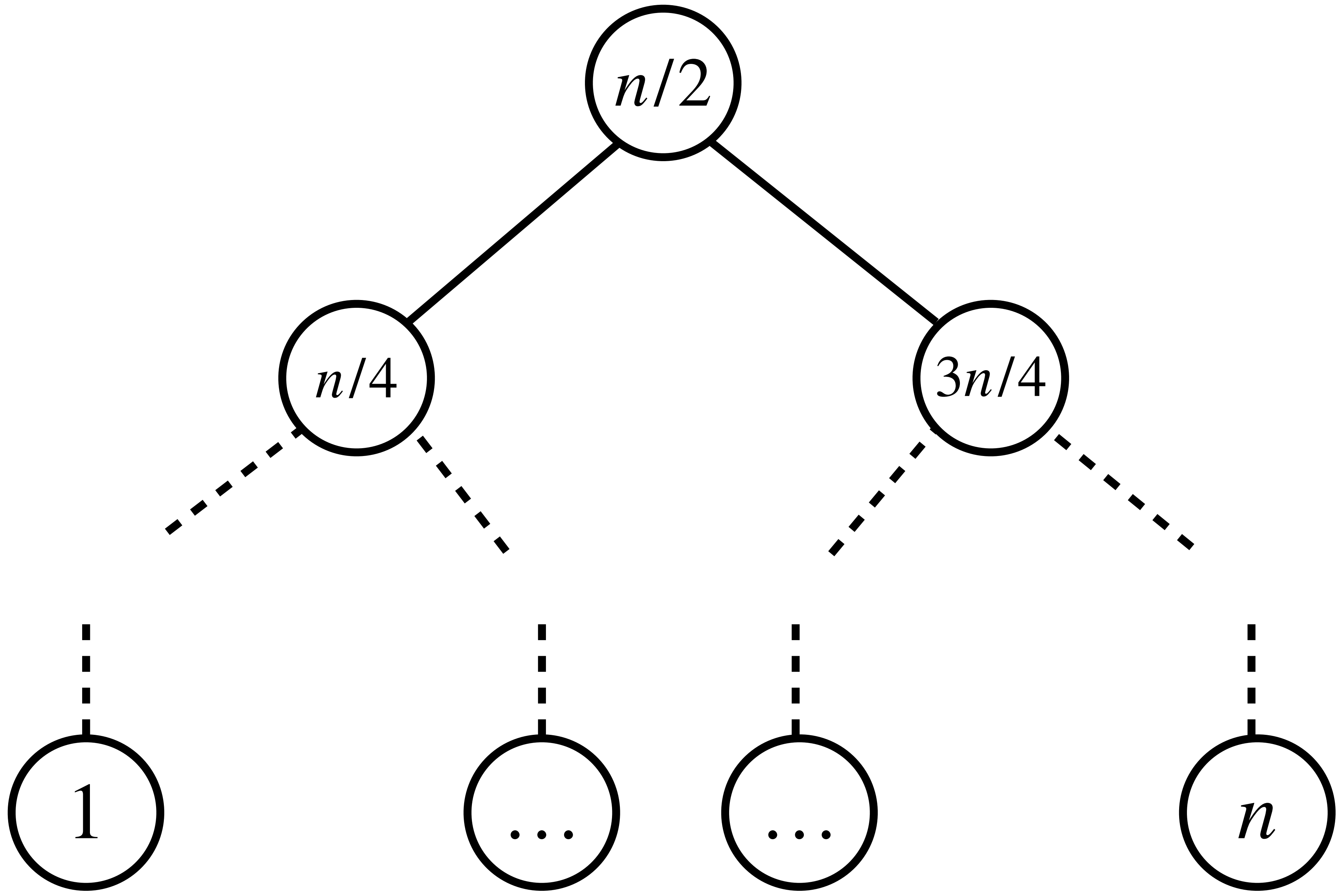

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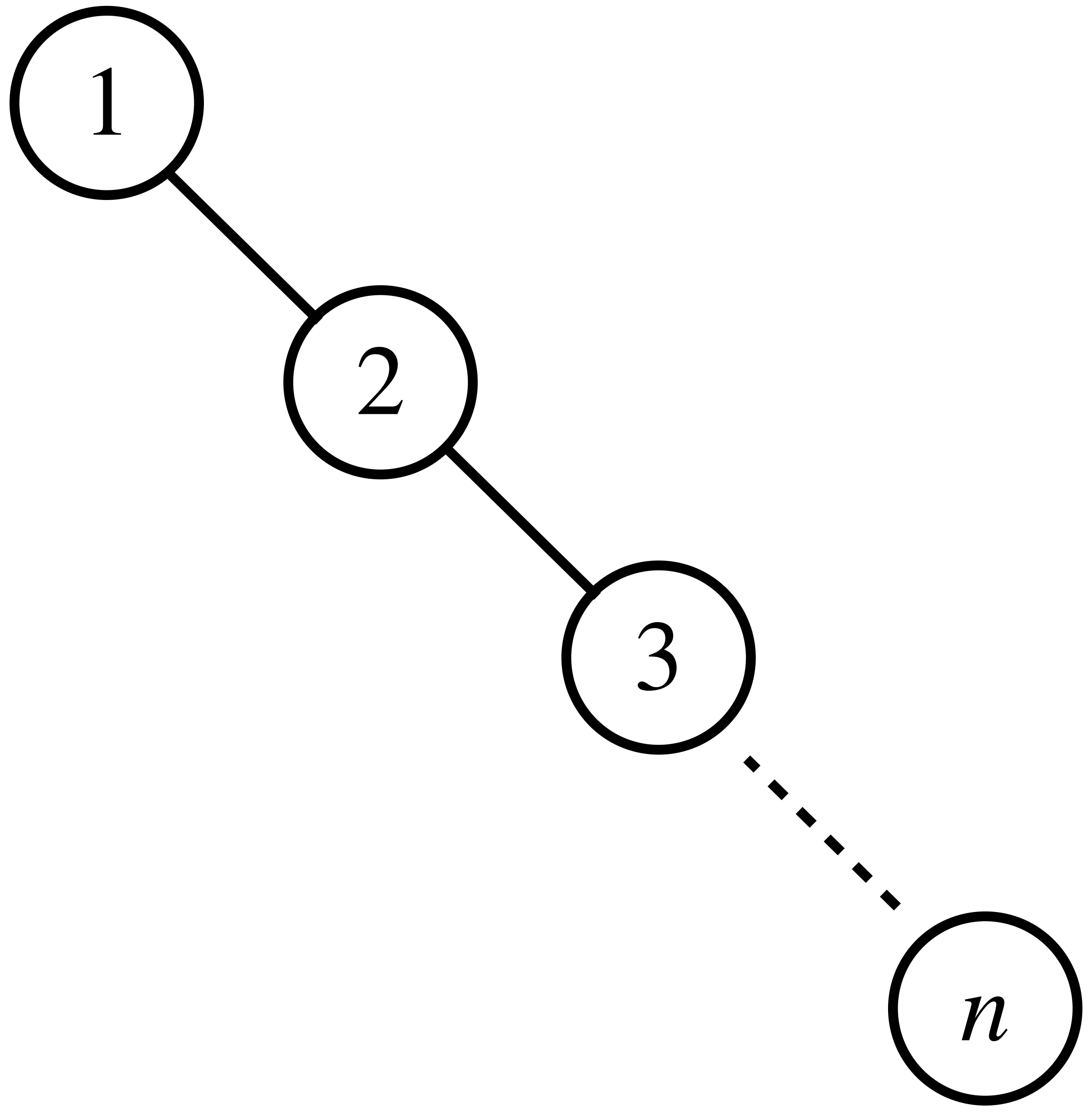

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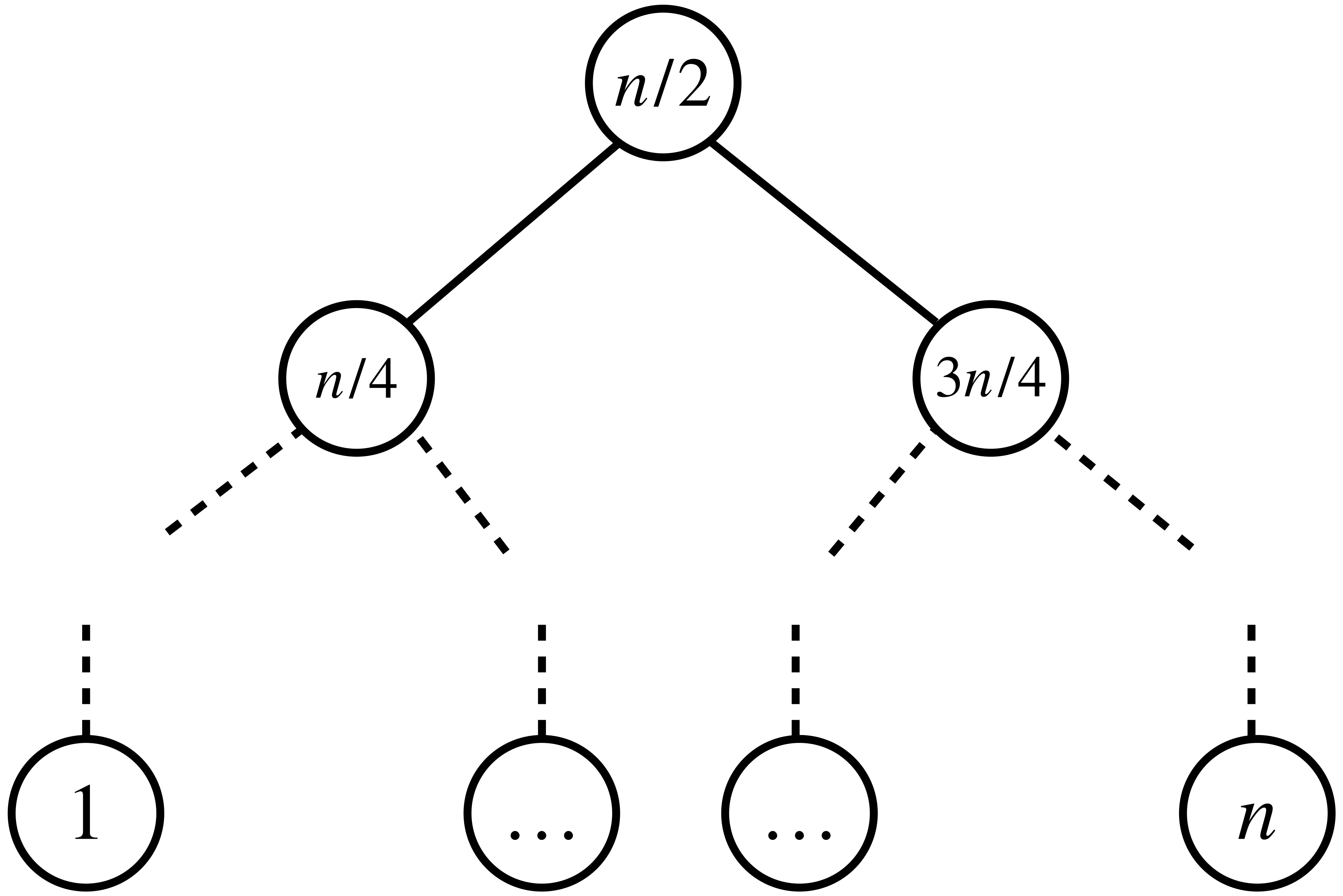

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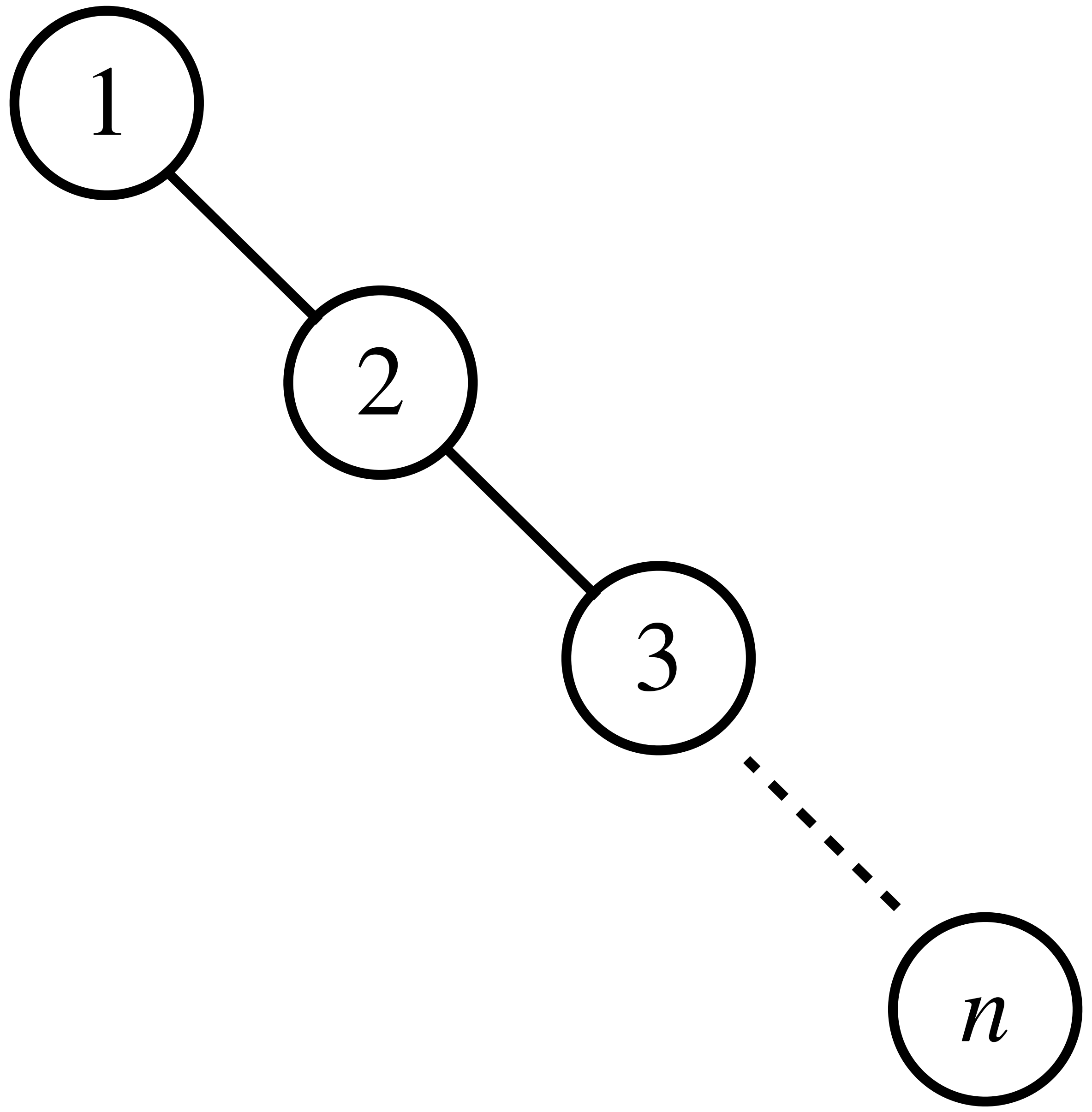

Worst case: $h = \Theta(n)$

How to Restrict Height in BSTs?

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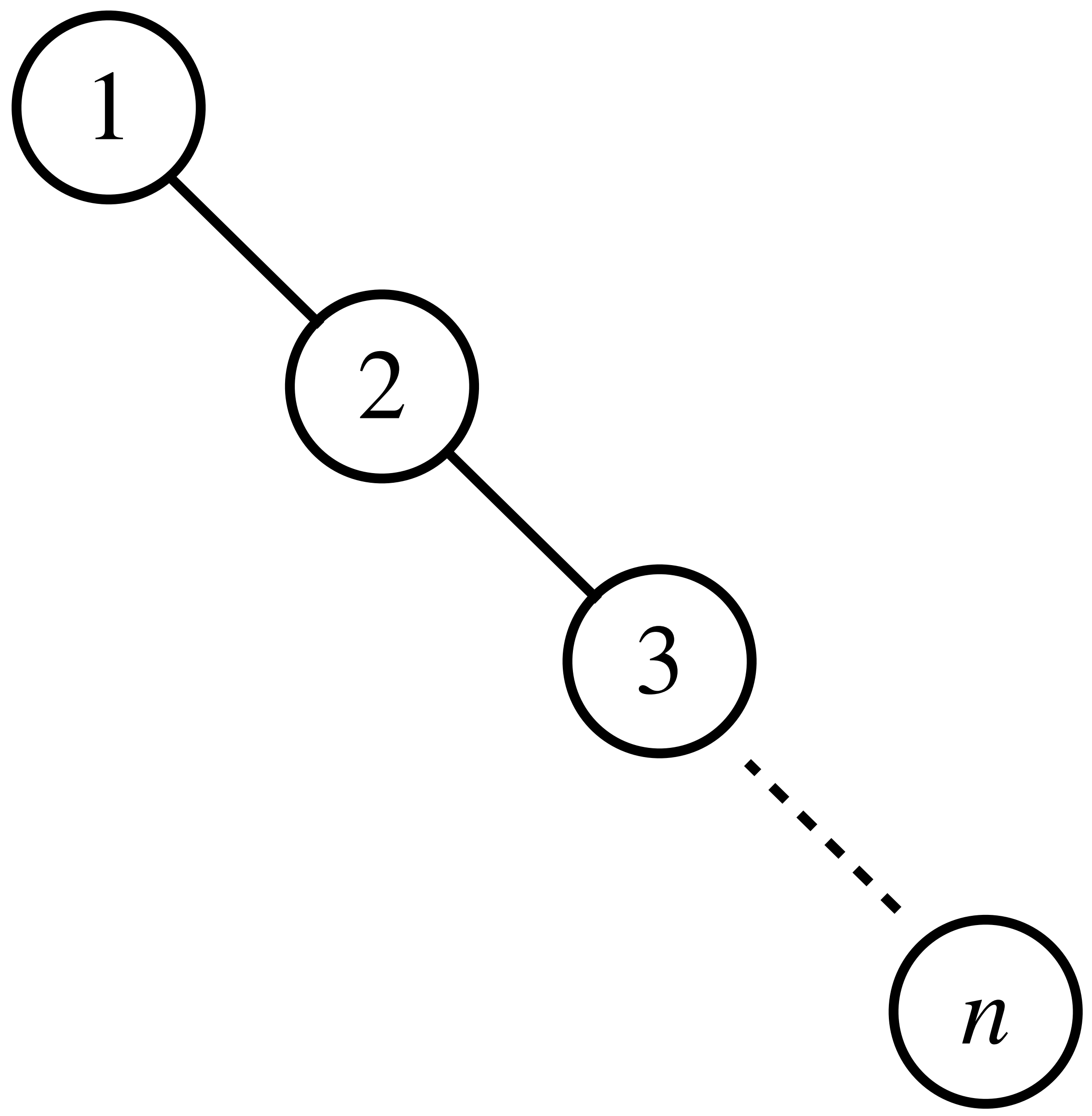

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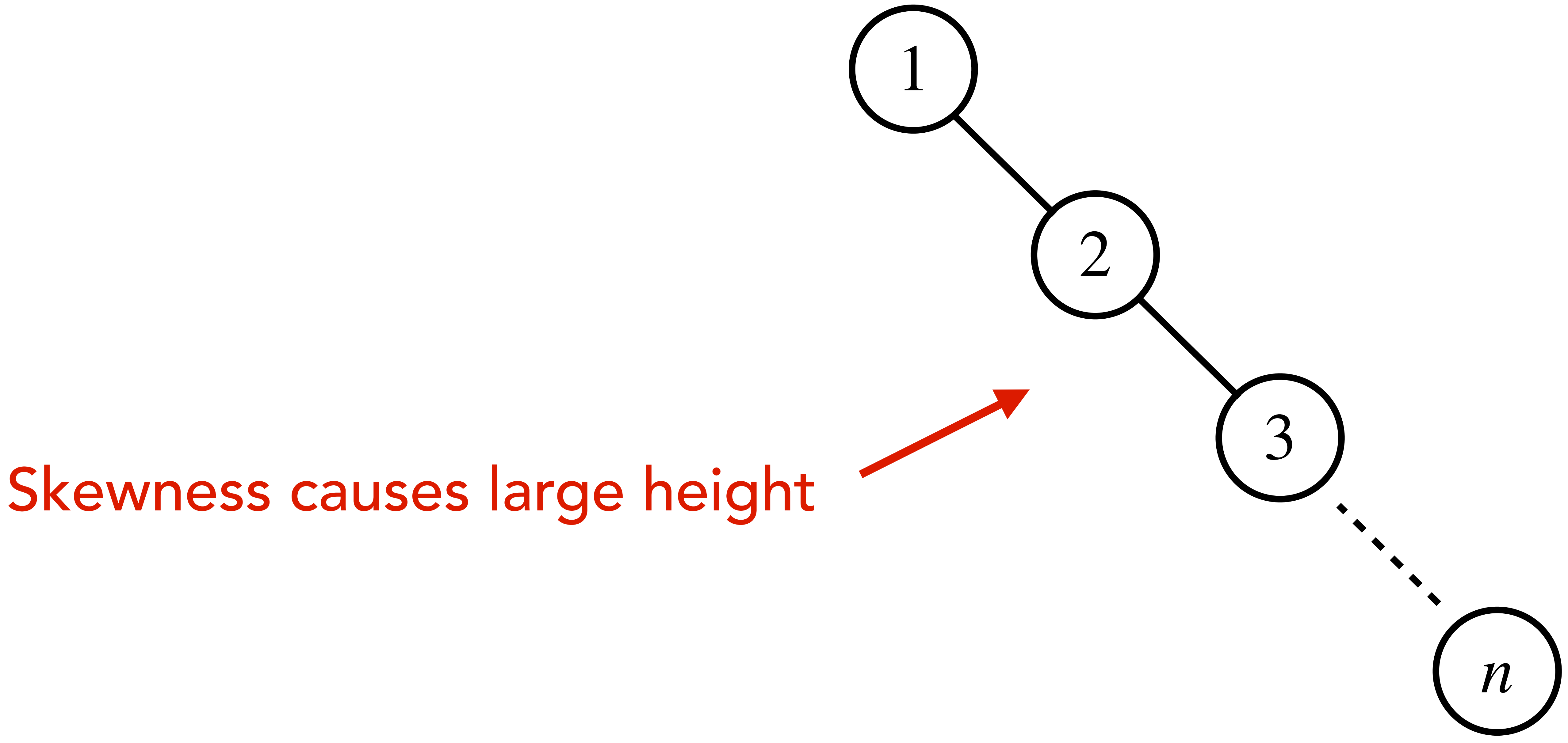

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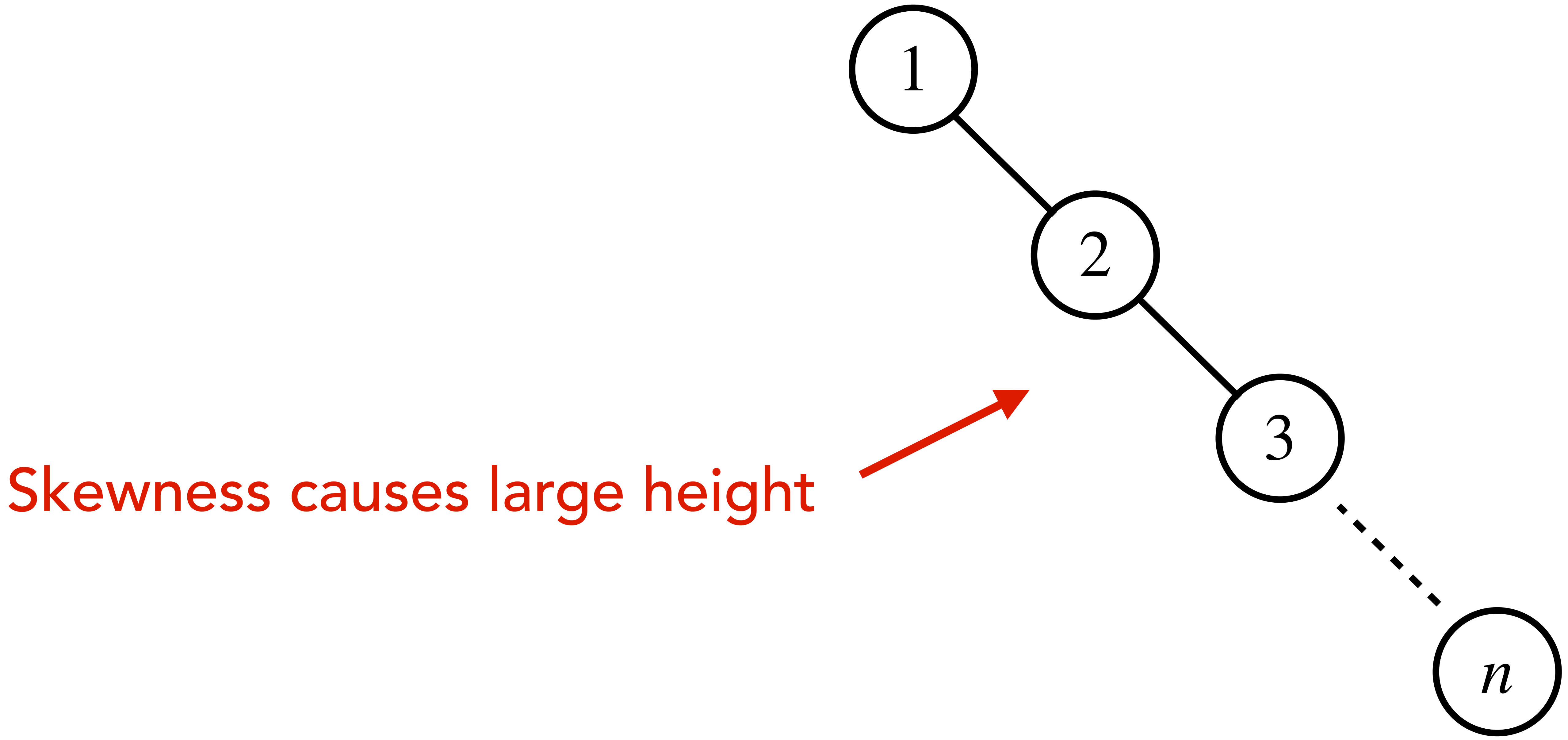

Idea: We can restrict the maximum height by keeping the BST balanced.

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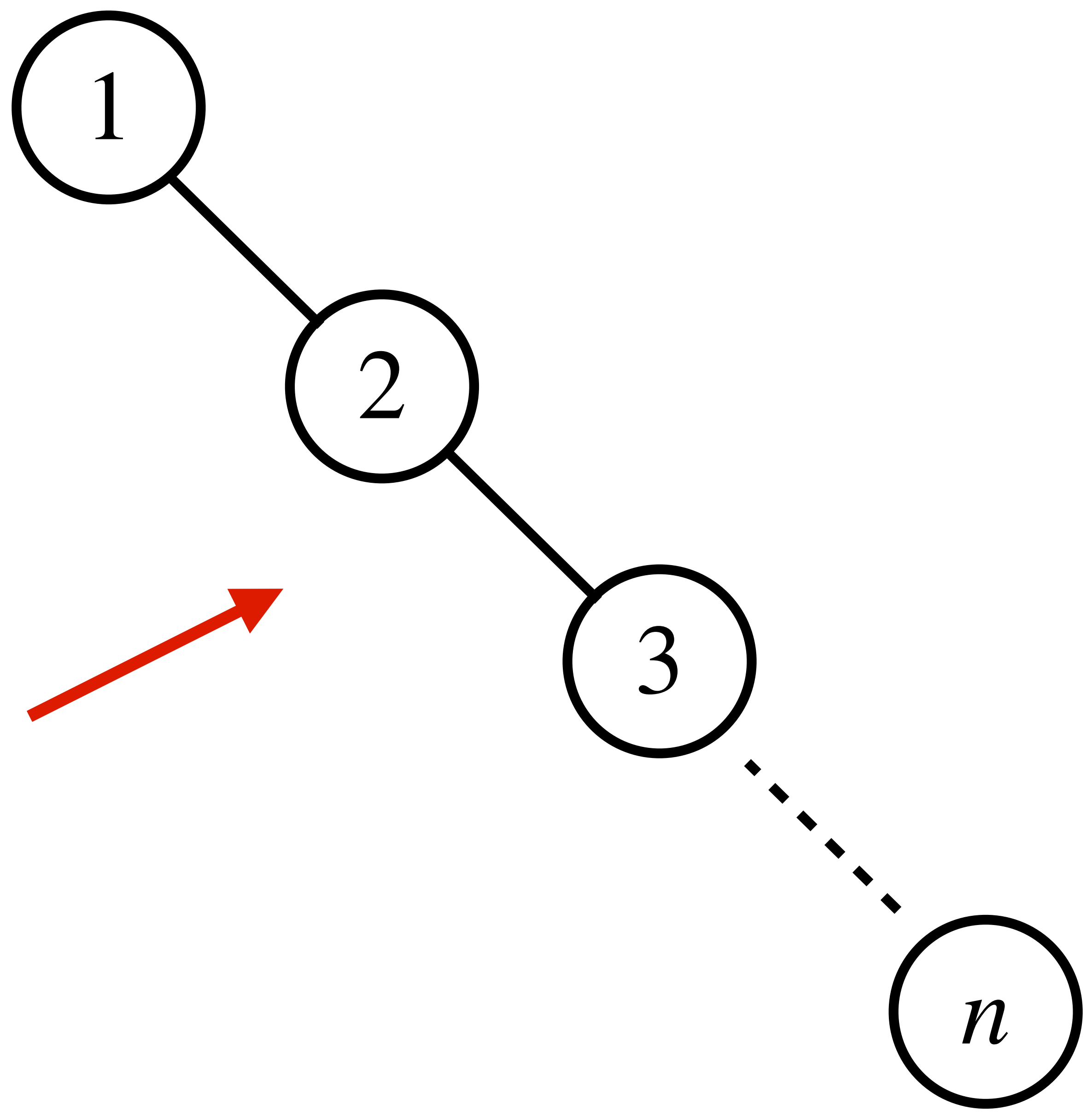

Skewness causes large height

Idea: We can restrict the maximum height by keeping the BST balanced.

For any node x, number of nodes in left-subtree(x) should not be too small or large than number of nodes in right-subtree(x)

RB-Trees: How do they look like?

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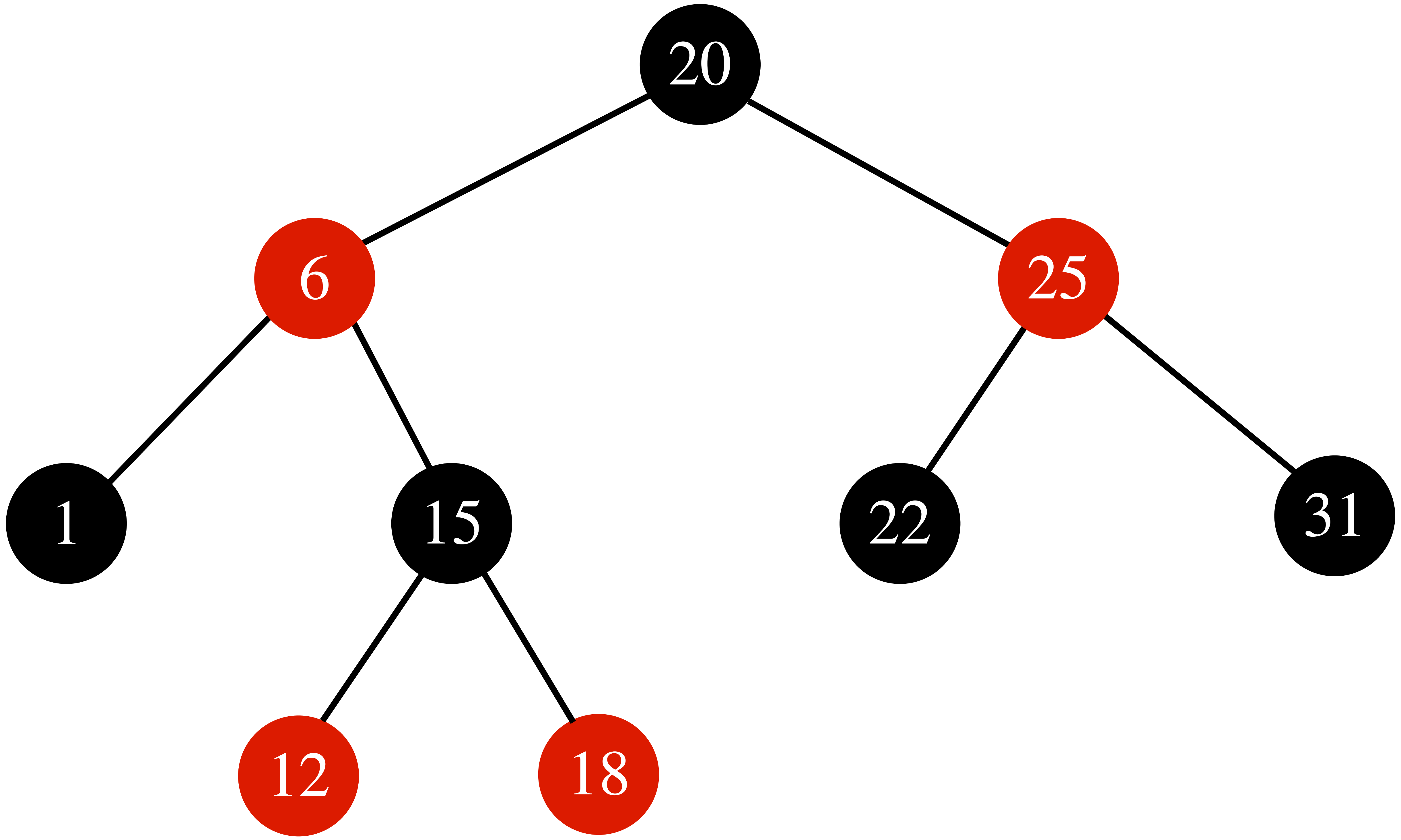

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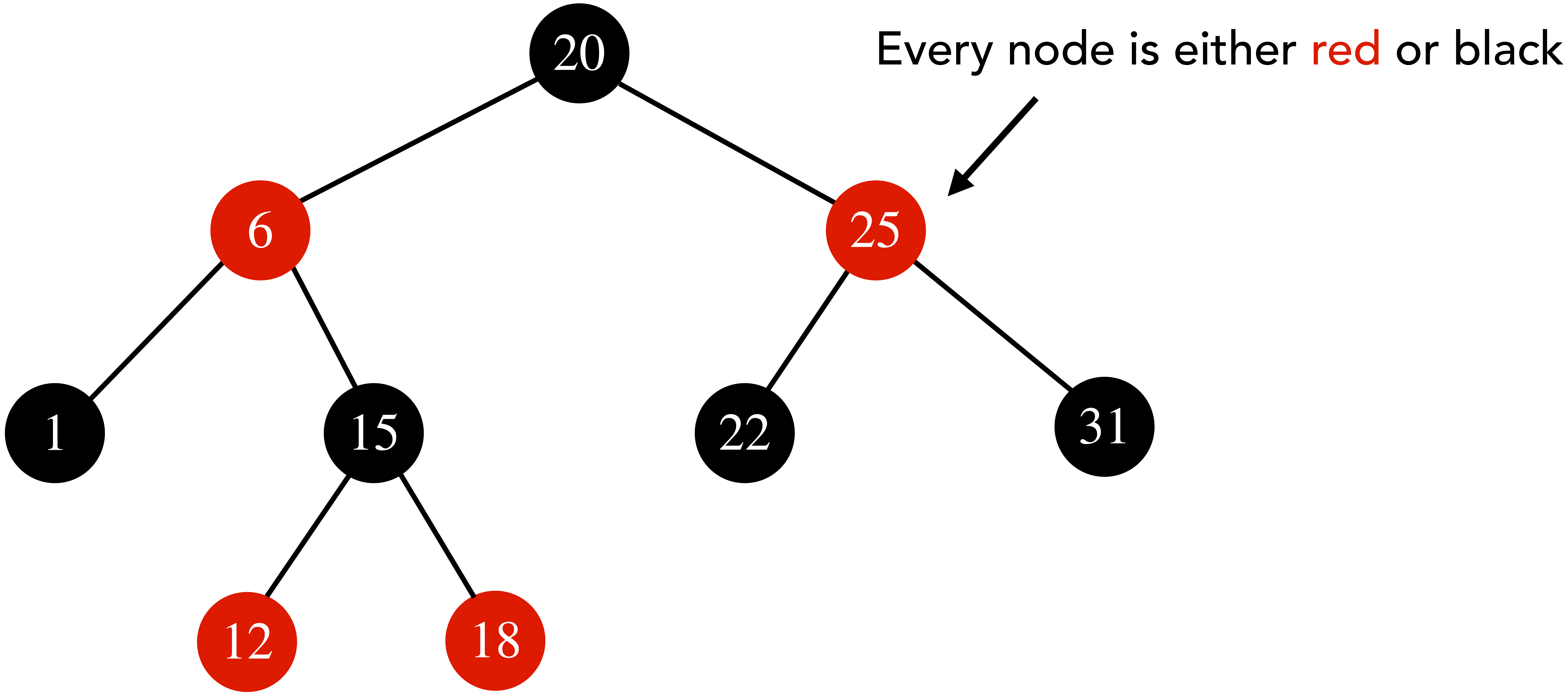

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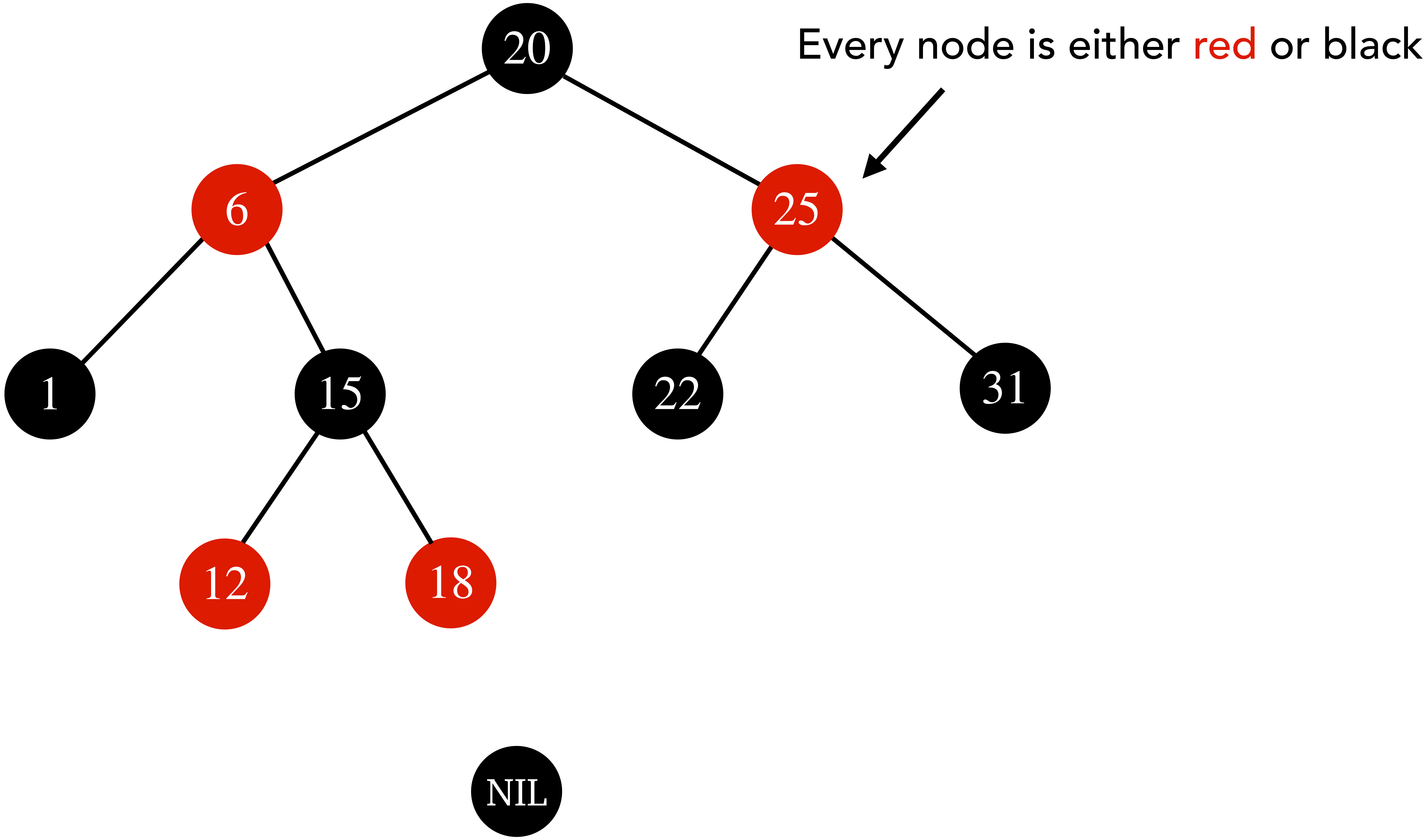

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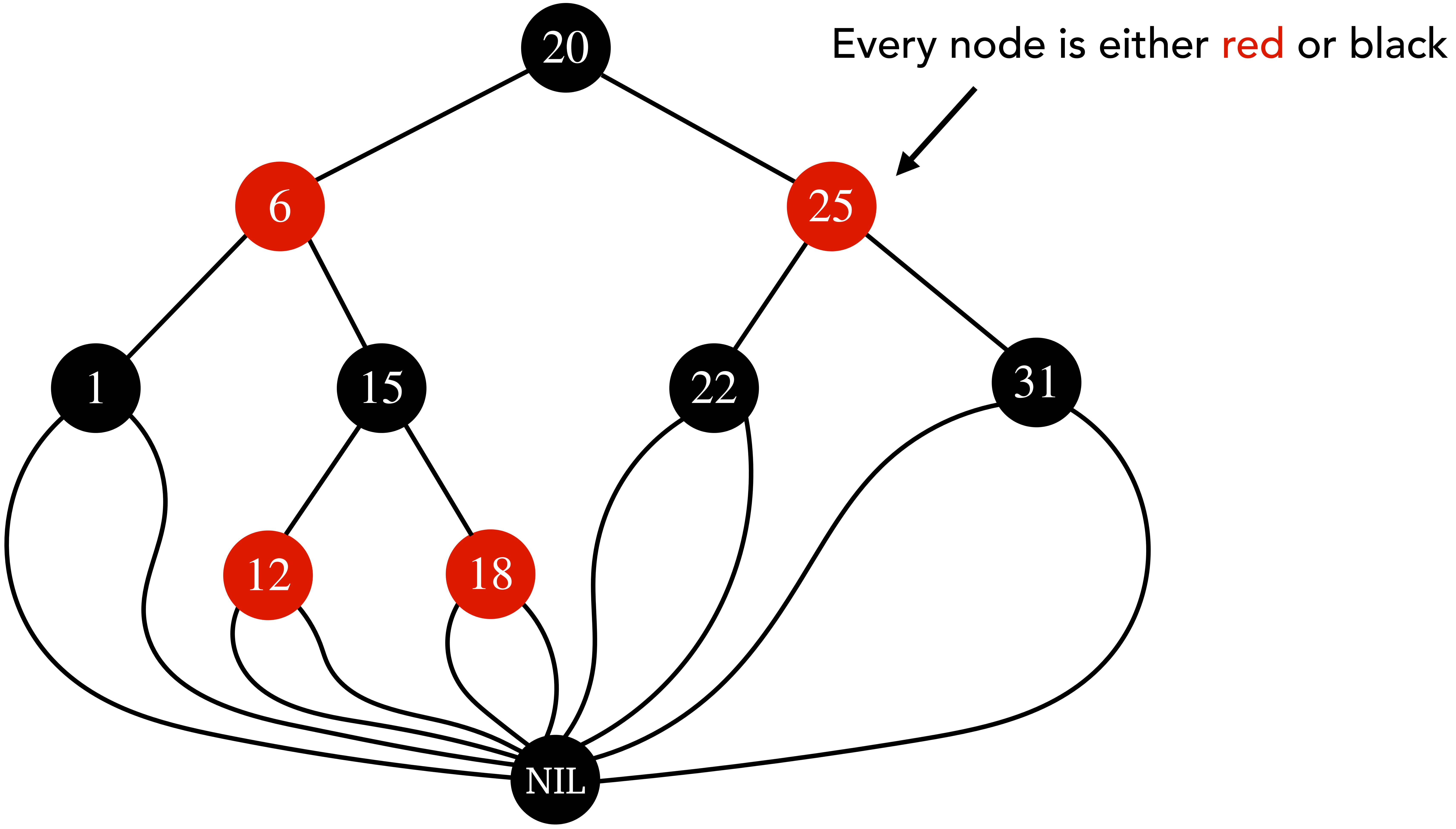

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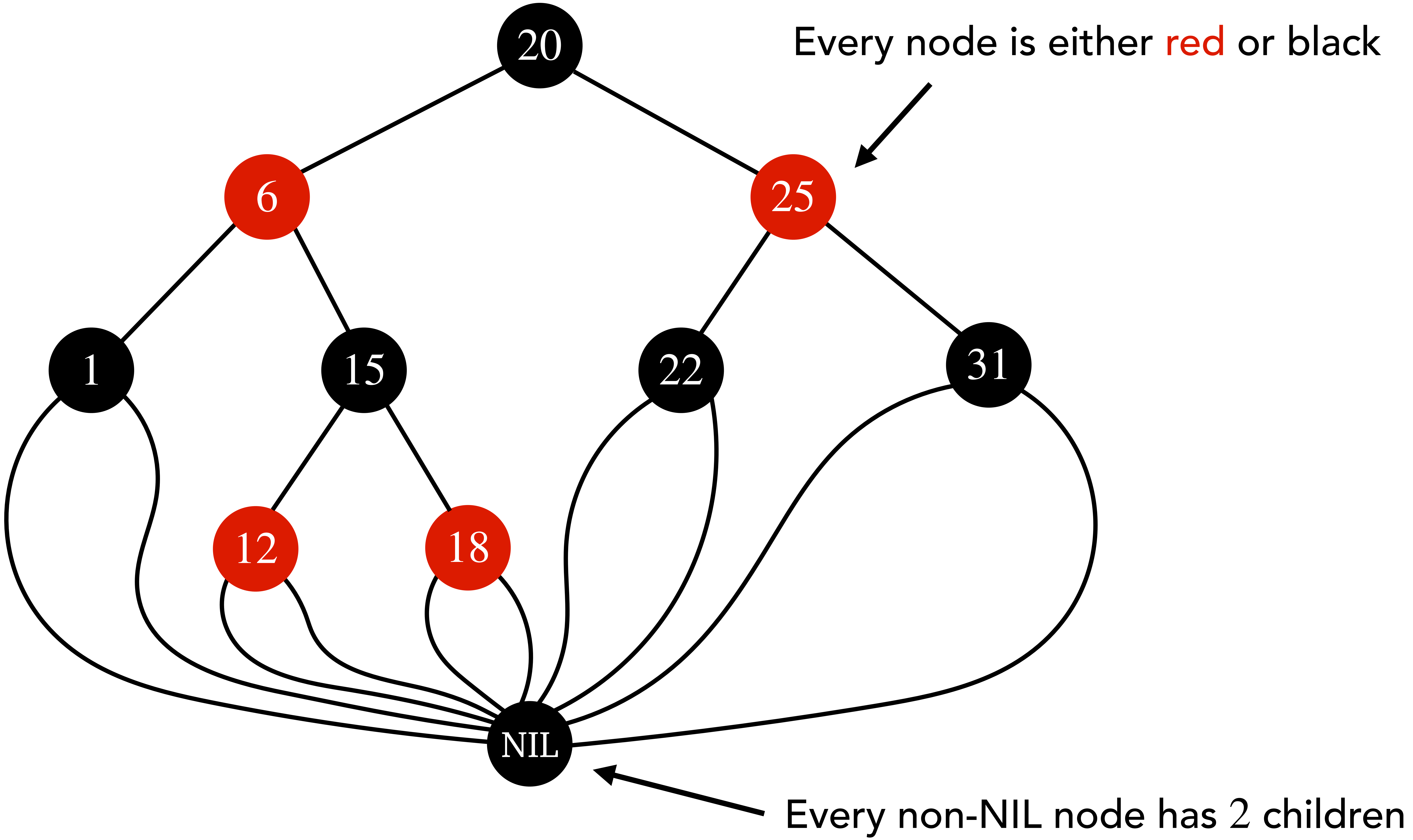

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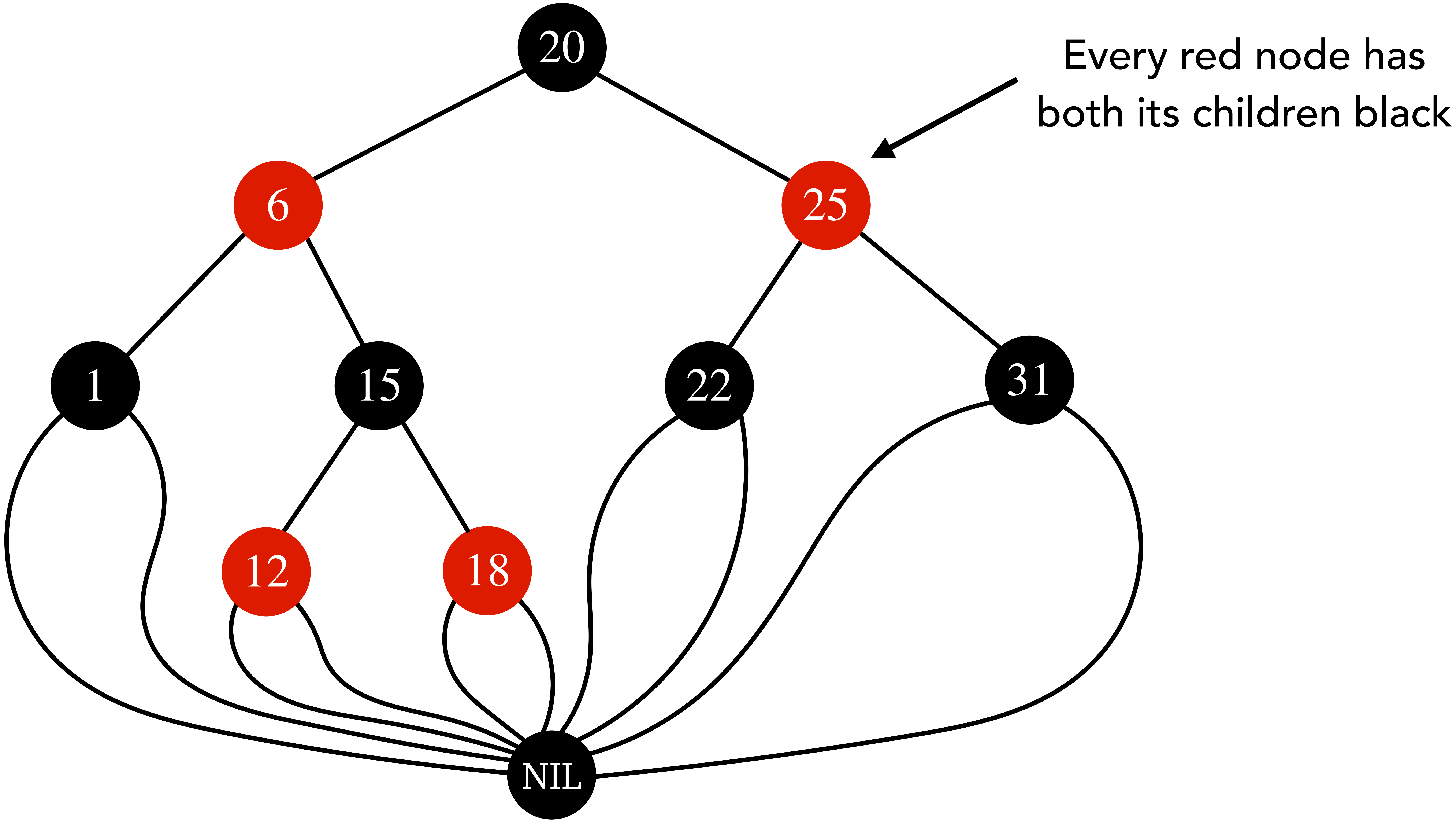

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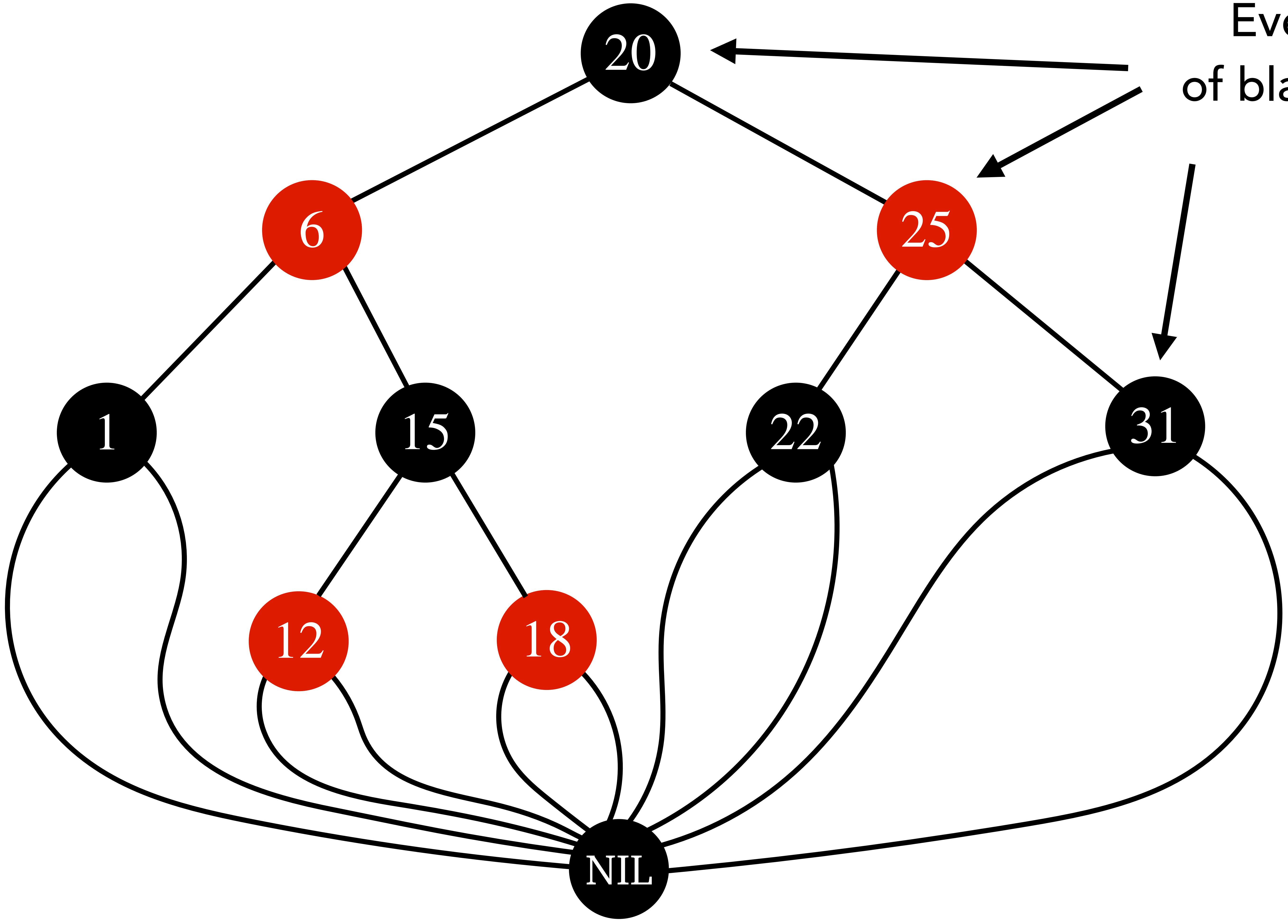

Every node has same number of black nodes on paths to leaves.

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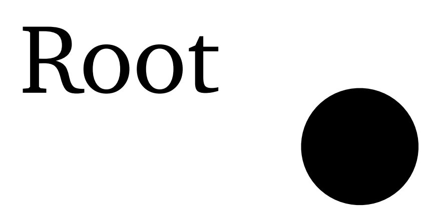

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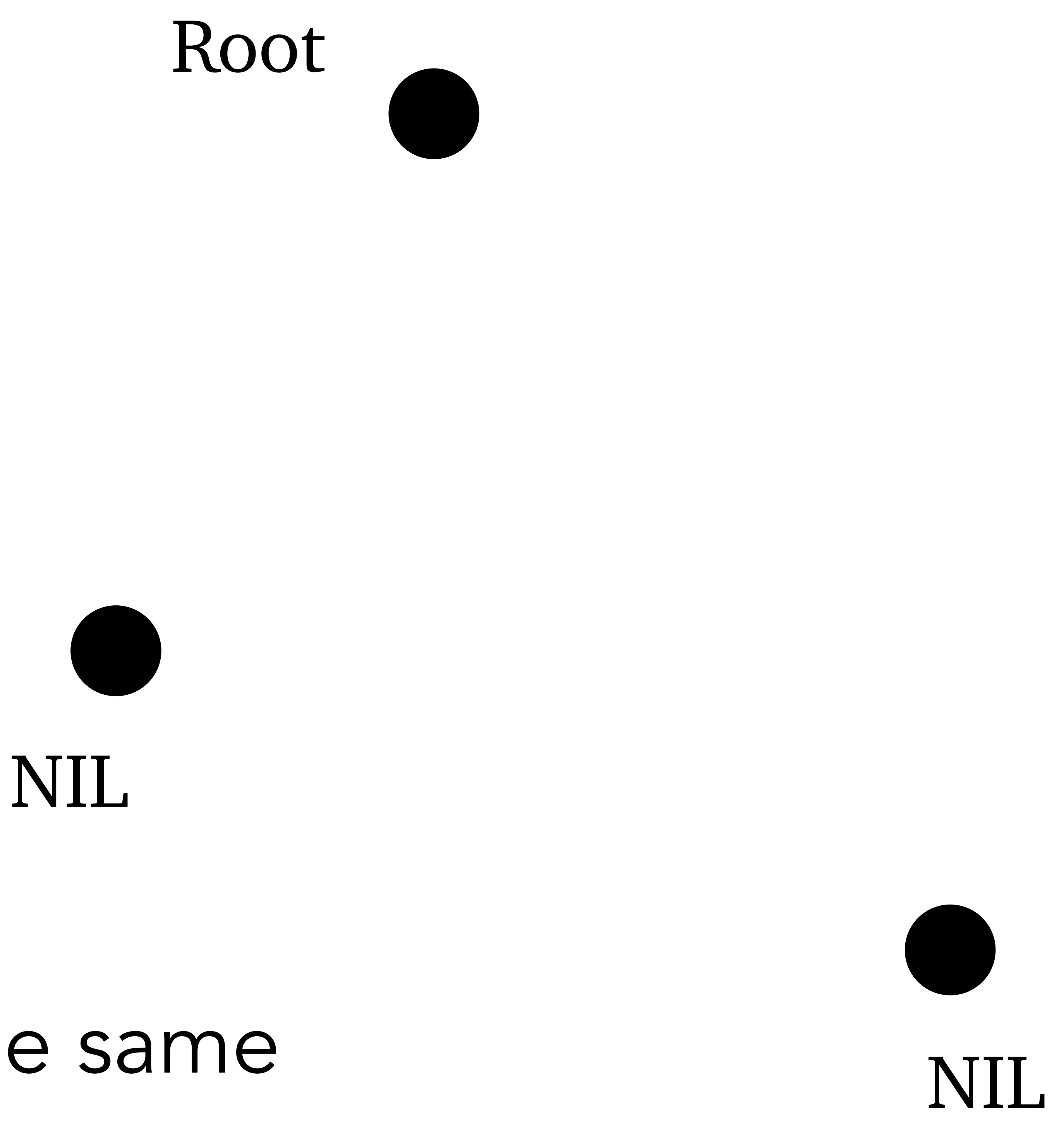

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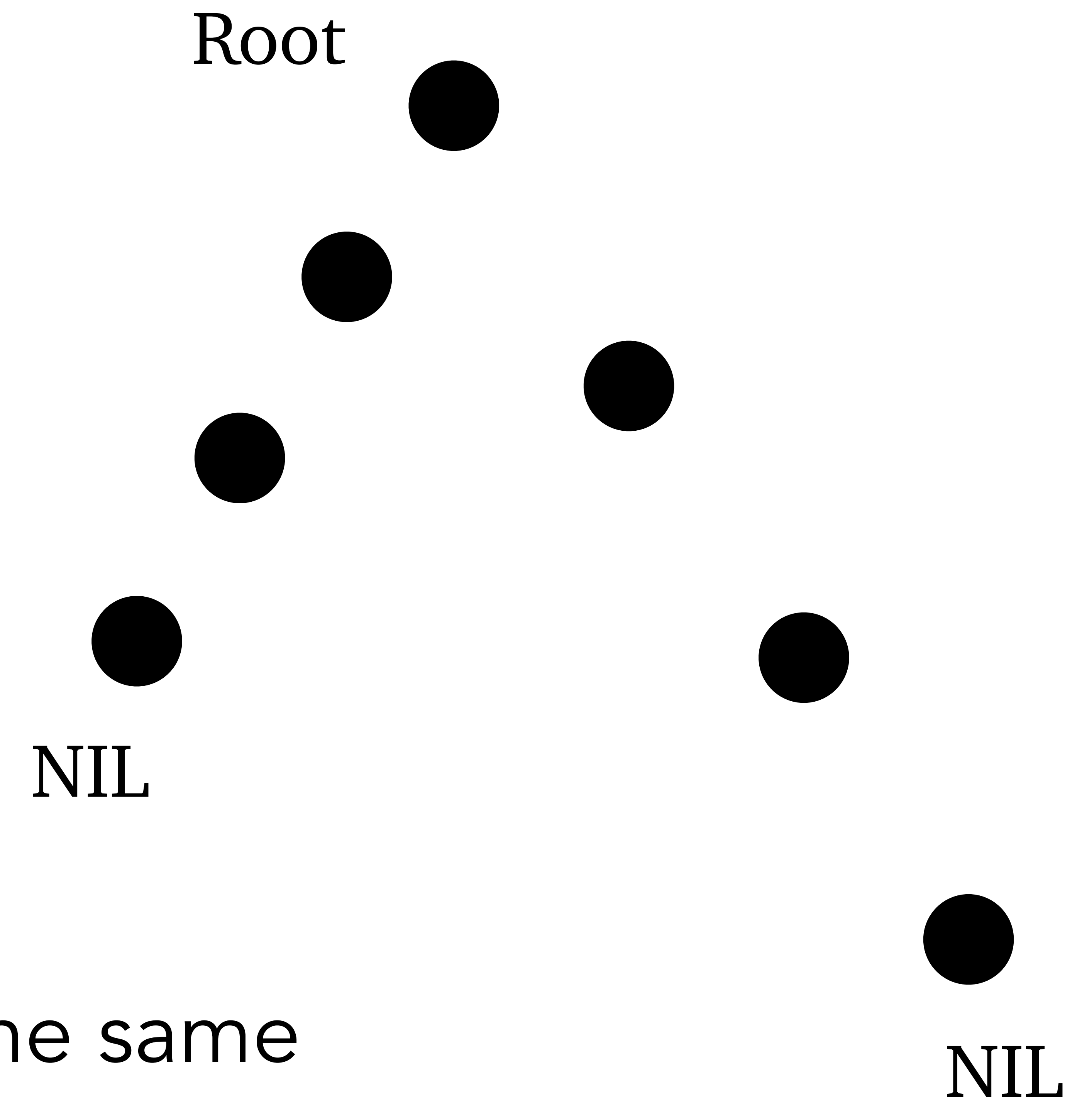

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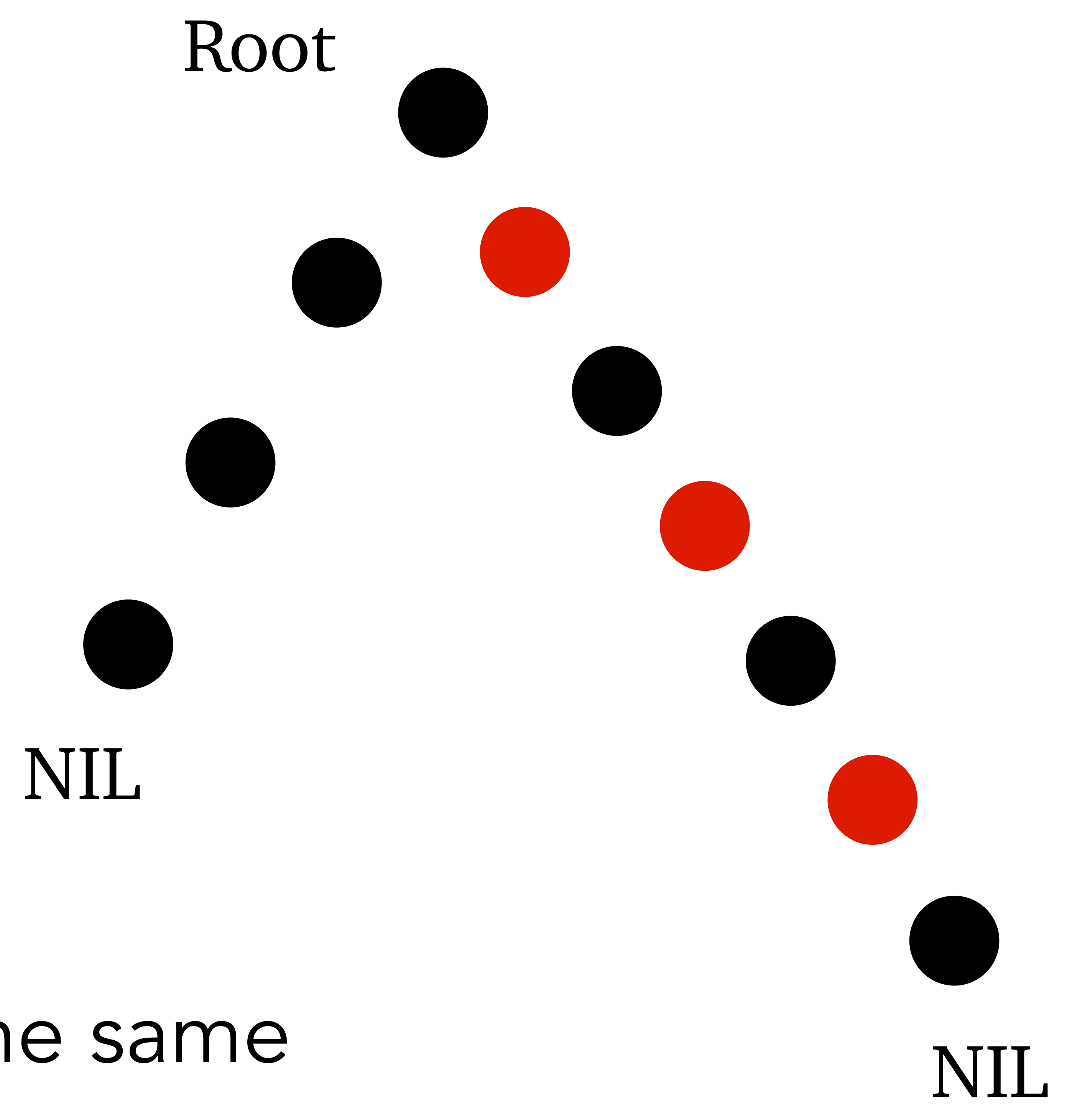

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Both these properties ensure that no path from root to a leaf is more than twice as long as any other.